Unified Framework for Linear MIMO Transceivers With Shaping Constraints

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Abstract—**This letter considers optimum linear transceivers for MIMO channels under a general framework based on Schur-concave and Schur-convex cost functions, subject to shaping constraints on the transmit covariance matrix. Such constraints may be useful, for example, to impose spectral masks in cable systems, to control the power transmitted along certain directions in wireless systems, or to limit the dynamic range of the power amplifiers at the different transmit dimensions.**

Index Terms—**Linear precoding, MIMO channel, peak power, spectral mask, transceiver.**

I. Introduction

The design of linear transceivers for multi-input multi-output (MIMO) channels (commonly referred to as linear precoders at the transmitter and equalizers at the receiver) has been considered in the literature according to different design criteria under an average power constraint (trace constraint) at the transmitter. The most used criterion is the minimization of the sum of the mean square error (MSE) of the channel substreams (the trace of the MSE matrix) [1]–[3].

Some other design criteria that have been considered include the minimization of the determinant of the MSE matrix [4], the maximization of the signal to interference-plus-noise ratio (SINR) with a zero-forcing (ZF) constraint [2], and the minimization of the bit error rate (BER) for ZF receivers [5]. In [6], a general framework that embraces the previous design criteria and generalizes upon them was developed based on Schur-convexity [7] (also with an average power constraint). In [8], some design criteria were considered with a peak power constraint (maximum eigenvalue constraint).

This letter considers optimum linear transceivers for MIMO channels under a general framework based on Schur-concave and Schur-convex cost functions, similar to that in [6], but subject to shaping constraints on the transmit covariance matrix rather than an average power constraint. Shaping constraints are useful to set limits on the shape of the transmitted power along different virtual directions. Some examples include: imposing spectral masks in cable systems (to control the crosstalk among users), limiting the power transmitted along certain directions in wireless systems (to avoid causing excessive interference), or limiting the dynamic range of the power amplifiers at the different transmit dimensions.

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II. Signal Model

Consider a MIMO communication channel with n_T transmit and n_R receive dimensions. The signal model is

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{n_T \times 1}$ is the transmitted vector, $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix, $\mathbf{y} \in \mathbb{C}^{n_R \times 1}$ is the received vector, and $\mathbf{n} \in \mathbb{C}^{n_R \times 1}$ is a zero-mean circularly symmetric complex Gaussian interference-plus-noise vector with arbitrary covariance matrix $\mathbf{R}_n$, i.e., $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_n)$.

With the use of a transmit linear processing matrix $\mathbf{B} \in \mathbb{C}^{n_T \times L}$ (commonly termed *linear precoder*), the transmitted vector can be written as

$$\mathbf{s} = \mathbf{B}\mathbf{x} \tag{2}$$

where $\mathbf{x} \in \mathbb{C}^{L \times 1}$ is the data vector that contains the L symbols to be transmitted. The total average transmitted power (in units of energy per transmission) is

$$P_T = \mathbb{E}[\|\mathbf{s}\|^2] = \mathrm{Tr}(\mathbf{B}\mathbf{B}^H) \tag{3}$$

where we have assumed zero-mean unit-energy uncorrelated symbols, i.e., $\mathbb{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{I}$.

Similarly, assuming a receive linear processing matrix $\mathbf{A}^H \in \mathbb{C}^{L \times n_R}$, the estimated data vector is

$$\hat{\mathbf{x}} = \mathbf{A}^H \mathbf{y}. \tag{4}$$

The main result of this letter only holds when the rank of the transmit covariance matrix $\mathbf{B}\mathbf{B}^H$ is unconstrained, i.e., when $L \geq n_T$ (typically, $L = n_T$). This is not a strong assumption and examples can be found in all kinds of systems: a single-input single-output (SISO) multicarrier system can transmit as many symbols as carriers, a wireline channel with L lines can clearly support L users, and a wireless multiantenna $n \times n$ channel can also support n substreams.

III. Shaping Constraints

A general shaping constraint on the transmit covariance matrix is of the form

$$\mathbf{B}\mathbf{B}^H \leq \mathbf{S}_B \tag{5}$$

where $\mathbf{S}_B$ is the upper bound (an equality constraint can be similarly considered) and $\mathbf{X} \geq \mathbf{Y}$ means that $\mathbf{X} - \mathbf{Y}$ is positive semidefinite. Such a shaping constraint includes as particular cases, among others, the following constraints.

A. Peak Power Constraint

Since $[\mathbf{BB}^H]_{ii} \leq \lambda_{\max}(\mathbf{BB}^H)$, one simple way to control the maximum power at each transmit dimension[1] is by imposing $\lambda_{\max}(\mathbf{BB}^H) \leq P_{\text{peak}}$ [9], [8] or, equivalently

$$\mathbf{BB}^H \leq P_{\text{peak}}\mathbf{I}. \tag{6}$$

This constraint is useful to control, for example, the dynamic range of the power amplifier at each transmit antenna [10].

B. Independent Power Constraint Per Transmit Dimension

One simple way to limit each diagonal element of the transmit covariance matrix $[\mathbf{BB}^H]_{ii} \leq p_i$ is

$$\mathbf{BB}^H \leq \operatorname{diag}\left(\{p_i\}_{i=1}^{n_T}\right). \tag{7}$$

The previous upper bound $\mathbf{S}_B$ is optimal in the sense that there is no other $\tilde{\mathbf{S}}_B \neq \mathbf{S}_B$ satisfying $[\tilde{\mathbf{S}}_B]_{ii} = p_i$ and $\tilde{\mathbf{S}}_B \geq \mathbf{S}_B$. There is, however, an infinite number of other choices with the same property (not comparable with $\tilde{\mathbf{S}}_B$); the considered one has the lowest condition number $\lambda_{\max}/\lambda_{\min}$ among such possible solutions.

C. Spectral Masks

In wireline systems, such as digital subscriber line (DSL), spectral masks are typically used to guarantee spectral compatibility among different users/services sharing the same cable [11]. Assuming a multicarrier modulation and modeling the system in a matrix form, the previously considered independent power constraint per transmit dimension is obtained.

D. Power Constraint Along a Direction

The power transmitted along the direction given by the unitary vector $\mathbf{u}$ is $\mathbf{u}^H\mathbf{BB}^H\mathbf{u}$. If the power along such a direction is to be limited to α, it suffices to choose a shaping upper bound $\mathbf{S}_B$ satisfying $\mathbf{u}^H\mathbf{S}_B\mathbf{u} \leq \alpha$. The same idea can be used to limit the power along several directions. This type of constraint may be very useful in multiuser scenarios when the interference caused to other users is to be limited (see, for example, [12] where EIRP constraints were considered).

IV. Optimum Transceivers With Shaping Constraints

Consider the following constrained optimization problem with a general shaping constraint on the transmit covariance matrix:

$$\begin{aligned} \min_{\mathbf{A},\mathbf{B}} \quad & f_0(\mathbf{d}(\mathbf{E}(\mathbf{B},\mathbf{A}))) \\ \text{s.t.} \quad & \mathbf{BB}^H \leq \mathbf{S}_B \end{aligned} \tag{8}$$

where $\mathbf{E}(\mathbf{B},\mathbf{A}) \triangleq \mathbb{E}[(\hat{\mathbf{x}}-\mathbf{x})(\hat{\mathbf{x}}-\mathbf{x})^H] = (\mathbf{A}^H\mathbf{HB}-\mathbf{I})(\mathbf{B}^H\mathbf{H}^H\mathbf{A}-\mathbf{I}) + \mathbf{A}^H\mathbf{R}_n\mathbf{A}$ is the MSE matrix, its diagonal elements $\mathbf{d}(\mathbf{E}(\mathbf{B},\mathbf{A}))$ are the MSEs of the different established substreams,[2] and f_0 is an arbitrary cost function of the MSEs (increasing in each of the arguments). In [6], a similar problem was considered with an average power constraint expressed as $\operatorname{Tr}(\mathbf{BB}^H) \leq P_T$.

In a similar way, cost functions of the SINRs and of the BERs could be considered. However, as shown in [6], such functions can always be expressed as functions of the MSEs due to the relation between the SINR and the MSE, and between the BER and the SINR (under the Gaussian approximation). Therefore, it suffices to consider cost functions of the MSEs without loss of generality (w.l.o.g.).

A. Optimal Receive Matrix

For a fixed transmitter $\mathbf{B}$, the optimum (linear) receiver is the MMSE receiver of Wiener filter [6]

$$\mathbf{A} = \mathbf{R}_n^{-1}\mathbf{HB}(\mathbf{I} + \mathbf{B}^H\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{HB})^{-1} \tag{9}$$

and the MSE matrix reduces then to

$$\mathbf{E} = (\mathbf{I} + \mathbf{B}^H\mathbf{R}_H\mathbf{B})^{-1} \tag{10}$$

where $\mathbf{R}_H \triangleq \mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}$. Note that the MMSE receiver is optimum regardless of f_0 since it minimizes simultaneously all MSEs (cf. [6]).

B. Optimal Transmit Matrix

Theorem 1: Consider the constrained optimization problem:

$$\begin{aligned} \min_{\mathbf{B}} \quad & f_0(\mathbf{d}((\mathbf{I} + \mathbf{B}^H\mathbf{R}_H\mathbf{B})^{-1})) \\ \text{s.t.} \quad & \mathbf{BB}^H \leq \mathbf{S}_B \end{aligned} \tag{11}$$

where matrix $\mathbf{B} \in \mathbb{C}^{n_T \times L}$ is the optimization variable $(L \geq n_T)$,[3] $\mathbf{R}_H \in \mathbb{C}^{n_T \times n_T}$ is a positive semidefinite Hermitian matrix, and $f_0 : \mathbb{R}^L \rightarrow \mathbb{R}$ is an arbitrary cost function (increasing in each variable).

It then follows that there is an optimal solution $\mathbf{B}$, such that $\mathbf{BB}^H = \mathbf{S}_B$, given by

$$\mathbf{B} = \mathbf{S}_B^{1/2}\mathbf{Q} \tag{12}$$

where $\mathbf{S}_B^{1/2} \in \mathbb{C}^{n_T \times L}$ is a "square-root" of $\mathbf{S}_B$ (not necessarily Hermitian or square) satisfying $\mathbf{S}_B^{1/2}\mathbf{S}_B^{H/2} = \mathbf{S}_B$ and $\mathbf{Q} \in \mathbb{C}^{L \times L}$ is a unitary matrix. The optimal solution and the minimum cost value are further simplified in the following cases.

- If f_0 is a Schur-concave function [7], [6], then the optimal $\mathbf{Q}$ has as columns the eigenvectors of $\mathbf{S}_B^{H/2}\mathbf{R}_H\mathbf{S}_B^{1/2}$ and the minimum cost value is given by
$$f_0\left(\boldsymbol{\lambda}\left(\left(\mathbf{I} + \mathbf{S}_B^{H/2}\mathbf{R}_H\mathbf{S}_B^{1/2}\right)^{-1}\right)\right).$$
- If f_0 is a Schur-convex function [7], [6], then the optimal $\mathbf{Q}$ is such that $\mathbf{Q}^H(\mathbf{I}+\mathbf{S}_B^{H/2}\mathbf{R}_H\mathbf{S}_B^{1/2})^{-1}\mathbf{Q}$ has equal diagonal elements (examples of matrices that satisfy this property can be obtained as the matrix with the eigenvectors of $\mathbf{S}_B^{H/2}\mathbf{R}_H\mathbf{S}_B^{1/2}$ (as in the previous case) multiplied by the unitary DFT matrix or the Hadamard matrix, cf., [6]) and the minimum cost value is given by
$$f_0\left(\mathbf{1}\frac{1}{L}\operatorname{Tr}\left(\left(\mathbf{I} + \mathbf{S}_B^{H/2}\mathbf{R}_H\mathbf{S}_B^{1/2}\right)^{-1}\right)\right).$$

Proof: See Appendix. ∎

Remarkably, the previous optimal solutions are independent of the specific choice of f_0 and only depend on whether f_0 is Schur-concave or Schur-convex (the optimal cost value does depend on f_0). The previous result not only tells that an optimal solution must satisfy $\mathbf{BB}^H = \mathbf{S}_B$ (which is an intuitive and expected result) but also characterizes the rotation $\mathbf{Q}$.

The maximization of the rate in a SISO multicarrier channel with a spectral mask is a particular case of Theorem 1 of a great

[1]It is a peak power constraint is in the sense of the peak along the transmit dimensions, but it is still an average power constraint.

[2]$\mathbf{d}(\mathbf{X})$ and $\boldsymbol{\lambda}(\mathbf{X})$ denote the vectors with the diagonal elements and eigenvalues of matrix $\mathbf{X}$, respectively.

[3]To be exact, it is only required that $L \geq \operatorname{rank}(\mathbf{S}_B)$.

practical interest in wireline systems. Such a problem was considered in [13] with results [13, eq. (5a)] in agreement with Theorem 1 (note that the total rate as measured with the gap-approximation method [13] is a Schur-concave function).

Corollary 1: For the particular case of a peak power constraint or eigenvalue constraint, expressed as $\mathbf{B}\mathbf{B}^H \leq P_{\text{peak}}\mathbf{I}$, the result of Theorem 1 simplifies to $\mathbf{B} = \sqrt{P_{\text{peak}}}[\mathbf{0}\ \mathbf{I}]\mathbf{Q}$. In this case, however, it is possible to relax the constraint $L \geq n_T$. In particular, the optimal solution for $L \leq n_T$ is then given by

$$\mathbf{B} = \sqrt{P_{\text{peak}}}\mathbf{U}_{H,1}\tilde{\mathbf{Q}} \tag{13}$$

where $\mathbf{U}_{H,1}$ contains as columns the L eigenvectors of $\mathbf{R}_H$ corresponding to the L largest eigenvalues in increasing order. To be more precise:

- if f_0 is a Schur-concave function, then $\tilde{\mathbf{Q}} = \mathbf{I}$;
- if f_0 is a Schur-convex function, then $\tilde{\mathbf{Q}}$ is, for example, the unitary DFT matrix or the Hadamard matrix.

For instance, if the trace or the determinant of the MSE matrix are to be minimized, an optimal transmit matrix (but not the only one) is simply given by $\mathbf{B} = \sqrt{P_{\text{peak}}}\mathbf{U}_{H,1}$ because the cost function is Schur-concave [6]. This result coincides with that obtained in [8] for a peak power constraint.

APPENDIX

First, rewrite the problem as

$$\begin{aligned}\min_{\mathbf{B},\tilde{\mathbf{S}}_B} \quad & f_0(\mathbf{d}((\mathbf{I}+\mathbf{B}^H\mathbf{R}_H\mathbf{B})^{-1})) \\ \text{s.t.} \quad & \mathbf{B}\mathbf{B}^H = \tilde{\mathbf{S}}_B \text{ and } \tilde{\mathbf{S}}_B \leq \mathbf{S}_B.\end{aligned}$$

It follows from $\mathbf{B}\mathbf{B}^H = \tilde{\mathbf{S}}_B$ that $\mathbf{B}$ can always be written as $\mathbf{B} = \tilde{\mathbf{S}}_B^{1/2}\mathbf{Q}$, where $\tilde{\mathbf{S}}_B^{1/2} \in \mathbb{C}^{n_T \times L}$ is any "square-root" of $\tilde{\mathbf{S}}_B$ (not necessarily Hermitian or square) satisfying $\tilde{\mathbf{S}}_B^{1/2}\tilde{\mathbf{S}}_B^{H/2} = \tilde{\mathbf{S}}_B$ and $\mathbf{Q} \in \mathbb{C}^{L\times L}$ is an arbitrary unitary matrix (note that since all square-roots are related by a unitary transformation, we can just consider one of them w.l.o.g.).

The problem can then be rewritten as

$$\begin{aligned}\min_{\tilde{\mathbf{S}}_B} \quad & \tilde{f}_0(\tilde{\mathbf{S}}_B) \\ \text{s.t.} \quad & \tilde{\mathbf{S}}_B \leq \mathbf{S}_B\end{aligned} \tag{14}$$

where

$$\tilde{f}_0(\tilde{\mathbf{S}}_B) = \min_{\mathbf{Q}} f_0\left(\mathbf{d}\left(\mathbf{Q}^H\left(\mathbf{I}+\tilde{\mathbf{S}}_B^{H/2}\mathbf{R}_H\tilde{\mathbf{S}}_B^{1/2}\right)^{-1}\mathbf{Q}\right)\right).$$

We now characterize the function $\tilde{f}_0(\tilde{\mathbf{S}}_B)$. Consider two $n\times n$ matrices $\mathbf{X}$ and $\mathbf{Y}$ satisfying $\mathbf{X} \geq \mathbf{Y}$. It then follows that $\mathbf{R}_H^{1/2}\mathbf{X}\mathbf{R}_H^{1/2} \geq \mathbf{R}_H^{1/2}\mathbf{Y}\mathbf{R}_H^{1/2}$, which implies [14, Cor. 7.7.4] that

$$\lambda_i\left(\mathbf{R}_H^{1/2}\mathbf{X}\mathbf{R}_H^{1/2}\right) \geq \lambda_i\left(\mathbf{R}_H^{1/2}\mathbf{Y}\mathbf{R}_H^{1/2}\right)$$

where $\lambda_i(\mathbf{X})$ is the ith eigenvalue of $\mathbf{X}$ in some specific order (e.g., decreasing or increasing). Now, since $\mathbf{X}$ and $\mathbf{Y}$ can always be written (as was done with $\tilde{\mathbf{S}}_B$) as $\mathbf{X} = \mathbf{X}^{1/2}\mathbf{Q}_X\mathbf{Q}_X^H\mathbf{X}^{H/2}$ and $\mathbf{Y} = \mathbf{Y}^{1/2}\mathbf{Q}_Y\mathbf{Q}_Y^H\mathbf{Y}^{H/2}$, respectively, and $\lambda_i(\mathbf{X}\mathbf{Y}) = \lambda_i(\mathbf{Y}\mathbf{X})$ for the nonzero eigenvalues [14, Th. 1.3.20], it follows that

$$\lambda_i(\mathbf{X}^{H/2}\mathbf{R}_H\mathbf{X}^{1/2}) \geq \lambda_i(\mathbf{Y}^{H/2}\mathbf{R}_H\mathbf{Y}^{1/2})$$

or, equivalently,

$$\lambda_i((\mathbf{I}+\mathbf{X}^{H/2}\mathbf{R}_H\mathbf{X}^{1/2})^{-1}) \leq \lambda_i((\mathbf{I}+\mathbf{Y}^{H/2}\mathbf{R}_H\mathbf{Y}^{1/2})^{-1}).$$

As a consequence, for any given $\mathbf{Q}$, there exists $\tilde{\mathbf{Q}}$ such that

$$\begin{aligned}&\mathbf{d}(\tilde{\mathbf{Q}}^H(\mathbf{I}+\mathbf{X}^{H/2}\mathbf{R}_H\mathbf{X}^{1/2})^{-1}\tilde{\mathbf{Q}}) \\ &\qquad \leq \mathbf{d}(\mathbf{Q}^H(\mathbf{I}+\mathbf{Y}^{H/2}\mathbf{R}_H\mathbf{Y}^{1/2})^{-1}\mathbf{Q})\end{aligned}$$

(simply note that if $\tilde{\boldsymbol{\lambda}} \geq \boldsymbol{\lambda}$, then $\mathbf{d}(\mathbf{Q}^H\text{diag}(\tilde{\boldsymbol{\lambda}})\mathbf{Q}) \geq \mathbf{d}(\mathbf{Q}^H\text{diag}(\boldsymbol{\lambda})\mathbf{Q})$ for any unitary matrix $\mathbf{Q}$). Finally, since the function f_0 is increasing in each argument, it follows that $\tilde{f}_0(\mathbf{X}) \leq \tilde{f}_0(\mathbf{Y})$ if $\mathbf{X} \geq \mathbf{Y}$. In other words, the function $\tilde{f}_0$ is S_+^n-decreasing.

The solution to problem (14) is now straightforward since $\tilde{f}_0$ is $\mathbf{S}_+^n$-decreasing in $\tilde{\mathbf{S}}_B$ and the problem is constrained by $\tilde{\mathbf{S}}_B \leq \mathbf{S}_B$. To be more specific, the minimum cost value is achieved when $\tilde{\mathbf{S}}_B = \mathbf{S}_B$ and is given by $\tilde{f}_0(\mathbf{S}_B)$. This is true because any other matrix $\tilde{\mathbf{S}}_B \neq \mathbf{S}_B$ satisfying the constraint $\tilde{\mathbf{S}}_B \leq \mathbf{S}_B$ is clearly worse than $\mathbf{S}_B$ in terms of $\tilde{f}_0$. Note that we cannot make a similar statement if $L < \text{rank}(\mathbf{S}_B)$, because there is an infinite number of matrices $\tilde{\mathbf{S}}_B$ with rank L (satisfying $\tilde{\mathbf{S}}_B \leq \mathbf{S}_B$) that are not comparable under the partial ordering defined by the positive semidefinite cone. In such a case, the only way to find the best matrix is to evaluate all of them with $\tilde{f}_0$ (whose result will depend on the particular channel realization $\mathbf{R}_H$ and no general claim can be made).

Finally, it remains to obtain a more explicit characterization of $\tilde{f}_0(\mathbf{S}_B)$ by finding the minimizing $\mathbf{Q}$. For this purpose, the results obtained in [6] can be directly used as we now restate without proof. If f_0 is a Schur-concave function, the optimum $\mathbf{Q}$ is such that $\mathbf{Q}^H\mathbf{S}_B^{H/2}\mathbf{R}_H\mathbf{S}_B^{1/2}\mathbf{Q}$ is diagonal, and, if f_0 is a Schur-convex function, the optimum $\mathbf{Q}$ is such that $\mathbf{Q}^H(\mathbf{I}+\mathbf{S}_B^{H/2}\mathbf{R}_H\mathbf{S}_B^{1/2})^{-1}\mathbf{Q}$ has equal diagonal elements.

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