Array Gain in the DMT Framework for MIMO Channels

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***Abstract*—Following the seminal work by Zheng and Tse on the diversity and multiplexing tradeoff (DMT) of multiple-input multiple-output (MIMO) channels, in this paper, we introduce the array gain to investigate the fundamental relation between transmission rate and reliability in MIMO systems. The array gain gives information on the power offset that results from exploiting channel state information at the transmitter or as a consequence of the channel model. Hence, the diversity, multiplexing, and array gain (DMA) analysis is able to cope with the limitations of the original DMT and provide an operational meaning in the sense that the DMA gains of a particular system can be directly translated into a parameterized characterization of its associated outage probability performance. In this paper, we derive the best DMA gains achievable by any scheme employing isotropic signaling in uncorrelated Rayleigh, semicorrelated Rayleigh, and uncorrelated Rician block-fading MIMO channels. We use these results to analyze the effect of important channel parameters on the outage performance at different points of the DMT curve.**

***Index Terms*—Array gain, diversity multiplexing tradeoff (DMT), outage probability, performance analysis of multiple-input multiple-output (MIMO) channels.**

I. Introduction

A. Benefits of MIMO Channels

MULTIPLE-INPUT multiple-output (MIMO) channels are well known to provide a number of benefits over conventional single-antenna (SISO) channels, which have been traditionally described by the diversity, multiplexing, and array (DMA) gains [1], [2].

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The diversity gain denotes the improvement in link reliability obtained by receiving replicas of the information signal through (ideally independent) fading links. With an increasing number of independent copies, the probability that at least one of the signals is not experiencing a deep fade increases, thereby improving the quality and reliability of reception. A general MIMO channel with n_T transmit and n_R receive antennas offers potentially $n_\mathsf{T} n_\mathsf{R}$ independently fading links and, hence, a maximum spatial diversity order of $n_\mathsf{T} n_\mathsf{R}$.

The multiplexing gain is responsible for MIMO systems offering a linear increase in the achievable data rate. MIMO channels admit the transmission of multiple independent data streams within the bandwidth of operation and, under suitable channel conditions, also the separation at the receiver. Each data stream experiences at least the same channel quality that would be experienced in an SISO channel, effectively enhancing the capacity by a multiplicative factor equal to the number of established streams. In general, the number of data streams that can be reliably supported by a MIMO channel coincides with the minimum between the number of transmit antennas n_T and the number of receive antennas n_R, i.e., $\min\{n_\mathsf{T}, n_\mathsf{R}\}$.

Finally, the array gain indicates the enhancement in received signal-to-noise ratio (SNR) that results from a coherent combining effect of the information signals. The coherent combining may be realized through spatial processing at the receive antenna array and/or spatial pre-processing at the transmit antenna array.

B. Fundamental Diversity and Multiplexing Tradeoff (DMT)

The design of MIMO communication schemes has been traditionally tackled to maximize the previous gains, especially the diversity or the multiplexing gain. Both design perspectives come from opposite ways of understanding the ever-present fading in wireless communications. On the one hand, when fading is considered a source of randomness that makes wireless links unreliable, the natural response is to use the multiple antennas for compensating random signal fluctuations and achieving a steady channel gain. The spatial dimension is used in this case to maximize diversity. Some examples of MIMO schemes fall within this category are space-time codes [3], [4] and orthogonal designs [5], [6]. A different line of thought suggests that fading can be beneficial through increasing the degrees of freedom available for communication [7], [8]. The resulting spatial multiplexing phenomenon was first exploited in [9] and by the BLAST and V-BLAST architectures [10]–[12]. This dichotomy in dealing with the fading process and, by extension, with the design and analysis of MIMO systems is, however, not appropriate. In fact, both diversity and multiplexing gains can be simultaneously obtained, but there is a tradeoff between how much of each type of gain any MIMO scheme can extract: higher spatial multiplexing comes at the price of sacrificing diversity.

This close relationship was foreseen and investigated in [13]–[16]. However, the DMT was established by Zheng and Tse in the excellent groundbreaking paper [17]. To be more specific, the work in [17] focuses on the high-SNR regime and provides the fundamental tradeoff curve achievable by any scheme, where the spatial multiplexing gain is understood as the fraction of capacity attained at high SNR and the diversity gain quantifies the high-SNR reliability of the system. Strategies maximizing independently the diversity or the multiplexing gain correspond to the two extreme points of the curve: maximum diversity with no multiplexing gain and maximum multiplexing gain with no diversity gain. The DMT curve bridges the gap between these two extremes and offers insights to understand the overall resources offered by MIMO channels.

The main problem with DMT framework is that the diversity gain, defined as the slope of the error probability curve in the high-SNR regime, provides only a coarse measure of performance, in the sense that it is unable to capture the impact of various relevant channel features and it is also insensitive to the presence of channel state information (CSI) at the transmitter [18]. Furthermore, it is difficult to translate any conclusion extracted from the DMT into the actual error probability of a particular scheme.

As a result, several attempts were made in the literature to endow the DMT with operational meaning. First, the diversity and multiplexing gains definitions were modified to hold for any finite SNR value, leading to a finite-SNR DMT [19], [20]. However, the derivations are based on a lower bound on the outage probability and the final results require an additional numerical optimization process, so that the simplicity of the original DMT is lost. Similarly, a finite-SNR DMT is also presented in [21]–[23] relaying on the Gaussian distribution of the outage probability in the large system limit. The authors point out the importance of the power offset when characterizing the outage probability and propose different multiplexing gain definitions to improve the convergence of the approximation. However, the Gaussian approximation fails to capture the tails of the distribution and limits the accuracy of these results to the low to moderate SNR regime.

A totally different approach is taken in [24], where the focus is again on the high-SNR regime but the notion of multiplexing gain is substituted by that of rate region to investigate scenarios in which the data rate does not scale linearly with the logarithm of the SNR as in [17]. To the best of authors' knowledge, this assumption hardly accommodates practical schemes. In any case, the throughput and reliability tradeoff in [24] is still independent of important parameters of the channel model.

C. Contributions

Here, we aim at completing the DMT framework by introducing the array gain in the picture while trying to keep the essence of the original formulation. That is, we use equivalent definitions of diversity and multiplexing gain to those in [17] and include a new performance indicator that is able to cope with the limitations of the DMT. The array gain, indeed, gives information on the power offset that results from exploiting CSI at the transmitter or as a consequence of the adopted channel model. The resulting DMA analysis provides then more insights into the fundamental relation between transmission rate and reliability in MIMO systems, since the error probability is now characterized by two parameters: diversity and array gains. In this sense, the DMA analysis is still a twofold tradeoff and must not be understood as a three-sided compromise between DMA gains.

In this paper, we present the best DMA gains achievable by any scheme, employing isotropic signaling in uncorrelated Rayleigh, semicorrelated Rayleigh, and uncorrelated Rician block-fading MIMO channels. The rest of this paper is organized as follows. Sections II-A and II-B introduce the channel and system models, respectively. Then, the DMA framework is formulated in Section II-C and derived in Section III for a strictly positive multiplexing gain, whereas the zero multiplexing point is investigated in Section IV. Finally, the main contribution of this paper is summarized in Section V.

II. Preliminaries and Problem Formulation

A. Channel Model

A MIMO channel with n_{T} transmit and n_{R} receive dimensions can be described by an $n_{\mathsf{R}} \times n_{\mathsf{T}}$ channel matrix $\mathbf{H}$, whose $(i,j)\mathrm{th}$ entry characterizes the propagation path between the $j\mathrm{th}$ transmit and the $i\mathrm{th}$ receive antenna. Usually, since there are a large number of scatters in the channel that contribute to the information signal at the receiver, the application of the central limit theorem results in Gaussian distributed channel matrix coefficients. Analogously to the single antenna channel, this model is referred to as MIMO Rayleigh or Rician fading channel, depending whether the channel entries are zero mean or not. More exactly, we assume that the channel matrix can be described as (see [25] and references therein)

$$\mathbf{H} = \sqrt{\frac{\mathsf{K}}{\mathsf{K}+1}}\mathbf{H}_{\mathsf{los}} + \sqrt{\frac{1}{\mathsf{K}+1}}\mathbf{\Sigma}_{\mathsf{R}}^{1/2}\mathbf{H}_{\mathsf{w}}\mathbf{\Sigma}_{\mathsf{T}}^{1/2} \tag{1}$$

where $\mathsf{K} \in [0,\infty)$ is a power normalization factor known as the Rician K-factor, $\mathbf{H}_{\mathsf{los}}$ is a deterministic $n_{\mathsf{R}} \times n_{\mathsf{T}}$ matrix containing the line-of-sight (LOS) components of the channel, $\mathbf{\Sigma}_{\mathsf{T}} = \left(\mathbf{\Sigma}_{\mathsf{T}}^{1/2}\right)\left(\mathbf{\Sigma}_{\mathsf{T}}^{1/2}\right)^{\dagger}$ and $\mathbf{\Sigma}_{\mathsf{R}} = \left(\mathbf{\Sigma}_{\mathsf{R}}^{1/2}\right)\left(\mathbf{\Sigma}_{\mathsf{R}}^{1/2}\right)^{\dagger}$ are the channel correlation matrices at the transmit and receive side, respectively, and $\mathbf{H}_{\mathsf{w}}$ is a random matrix with i.i.d. zero-mean unit-variance circularly symmetric Gaussian entries. For a fair comparison of the different cases, the total average received power is assumed to be constant and, hence, we can impose without loss of generality that $\mathrm{tr}\,(\mathbf{\Sigma}_{\mathsf{T}}) = n_{\mathsf{T}}$, $\mathrm{tr}\,(\mathbf{\Sigma}_{\mathsf{R}}) = n_{\mathsf{R}}$, and $\mathrm{tr}\left(\mathbf{H}_{\mathsf{los}}\mathbf{H}_{\mathsf{los}}^{\dagger}\right) = n_{\mathsf{R}}n_{\mathsf{T}}$. In this paper, we consider the following important particular cases of the general channel model in (1).

Definition 1: The uncorrelated Rayleigh MIMO fading channel model is defined as

$$\mathbf{H} = \mathbf{H}_{\mathsf{w}} \tag{2}$$

where $\mathbf{H}_{\mathsf{w}}$ is an $n_{\mathsf{R}} \times n_{\mathsf{T}}$ random matrix with i.i.d. zero-mean unit-variance complex Gaussian entries.

Definition 2: The semicorrelated Rayleigh fading MIMO channel model with correlation at the side with minimum number of antennas[1] is defined as

$$\mathbf{H} = \begin{cases} \mathbf{\Sigma}^{1/2}\mathbf{H}_{\mathsf{w}} & n_{\mathsf{R}} \leq n_{\mathsf{T}} \\ \mathbf{H}_{\mathsf{w}}\mathbf{\Sigma}^{1/2} & n_{\mathsf{R}} > n_{\mathsf{T}} \end{cases} \tag{3}$$

where $\mathbf{\Sigma} = \left(\mathbf{\Sigma}^{1/2}\right)\left(\mathbf{\Sigma}^{1/2}\right)^{\dagger}$ is the $n \times n$ positive-definite channel correlation matrix with eigenvalues $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$ ordered such that $\sigma_1 > \cdots > \sigma_n > 0$, $n = \min(n_{\mathsf{T}}, n_{\mathsf{R}})$, and $\mathbf{H}_{\mathsf{w}}$ is defined in (2).

Definition 3: The uncorrelated Rician fading MIMO channel model is defined as

$$\mathbf{H} = \sqrt{\frac{\mathsf{K}}{\mathsf{K}+1}}\mathbf{H}_{\mathsf{los}} + \sqrt{\frac{1}{\mathsf{K}+1}}\mathbf{H}_{\mathsf{w}} \tag{4}$$

where $\mathsf{K} \in (0, \infty)$ is the Rician factor, $\mathbf{H}_{\mathsf{los}}$ is an $n_{\mathsf{R}} \times n_{\mathsf{T}}$ deterministic matrix containing the LOS components of the channel, and $\mathbf{H}_{\mathsf{w}}$ is defined in (2). For convenience, we introduce the noncentrality matrix $\mathbf{\Omega}$ defined as

$$\mathbf{\Omega} = \begin{cases} \mathsf{K}\mathbf{H}_{\mathsf{los}}\mathbf{H}_{\mathsf{los}}^{\dagger} & n_{\mathsf{R}} \leq n_{\mathsf{T}} \\ \mathsf{K}\mathbf{H}_{\mathsf{los}}^{\dagger}\mathbf{H}_{\mathsf{los}} & n_{\mathsf{R}} > n_{\mathsf{T}} \end{cases} \tag{5}$$

with eigenvalues $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_n)$ ordered such that $\omega_1 > \cdots > \omega_n > 0$.

B. System Model

We consider a wireless communication system with n_{T} transmit and n_{R} receive antennas, in which the channel matrix $\mathbf{H}$ is drawn from one of the channel models presented in Section II-A and remains constant within a block of n_{S} symbols after which it changes to an independent realization. In this situation, the received signal within one block can be gathered in an $n_{\mathsf{R}} \times n_{\mathsf{S}}$ matrix $\mathbf{Y}$ related to the $n_{\mathsf{T}} \times n_{\mathsf{S}}$ transmitted matrix $\mathbf{X}$ as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W} \tag{6}$$

where $\mathbf{W}$ is the additive white Gaussian noise and has i.i.d. entries with zero mean and unit variance. The transmitted signal $\mathbf{X}$ is normalized forcing the transmit power per block to satisfy

$$\frac{1}{n_{\mathsf{S}}}\mathbb{E}\left\{\|\mathbf{X}\|_{\mathrm{F}}^2\right\} \leq \mathsf{snr} \tag{7}$$

where snr is the average SNR at each receive antenna. In addition, the instantaneous CSI is assumed to be perfectly known at the receiver.

Under such a system setup, the outage probability is the primary measure of interest in the sense that it is the best achievable block error probability (BLER) in the limit of large codeword length [26], [27]. The outage probability is defined as the infimum of the probability that the instantaneous mutual information falls below the transmission rate R and, for the system model in (6), is given by [17, Sec. III-B]

$$\mathsf{P}_{\mathsf{out}}(\mathsf{R}, \mathsf{snr}) = \inf_{\mathbf{Q}\geq 0, \mathrm{tr}(\mathbf{Q})\leq \mathsf{snr}} \mathsf{Pr}\left(\log\det(\mathbf{I}_{n_{\mathsf{R}}} + \mathbf{H}\mathbf{Q}\mathbf{H}^{\dagger}) < \mathsf{R}\right). \tag{8}$$

[1]When, in contrast to Definition 2, we have correlation at the side with maximum number of antennas, the joint distribution of the channel eigenvalues is slightly different, but the proof techniques employed here for the min-semicorrelated channel model apply verbatim to the max-semicorrelated channel model (see [25] for details).

C. DMA Analysis Formulation

Recall that our main objective is to complete the DMT framework by including the array gain in order to provide additional information on the system performance. Hence, let us first formalize the concepts of diversity, multiplexing, and array gains.

As in [17], we define a scheme as a family of codes $\{\mathcal{C}(\mathsf{snr})\}$ of block length n_{S}, which employs a different code $\mathcal{C}(\mathsf{snr})$ with rate $\mathsf{R}(\mathsf{snr})$ for each SNR level. Then, a MIMO coding scheme $\{\mathcal{C}(\mathsf{snr})\}$ is said to achieve a spatial multiplexing gain r, a diversity gain $\mathsf{d}(\mathsf{r})$, and an array gain $\mathsf{a}(\mathsf{r})$ if the data rate is such that

$$\lim_{\mathsf{snr}\to\infty} \frac{\mathsf{R}(\mathsf{snr})}{\log \mathsf{snr}} = \mathsf{r} \tag{9}$$

where $0 \leq \mathsf{r} \leq \min(n_{\mathsf{T}}, n_{\mathsf{R}})$ and the outage probability satisfies[2]

$$\lim_{\mathsf{snr}\to\infty} \frac{\log \mathsf{P}_{\mathsf{out}}(\mathsf{r}, \mathsf{snr})}{\log \mathsf{snr}} = -\mathsf{d}(\mathsf{r}) \tag{10}$$

$$\lim_{\mathsf{snr}\to\infty} \frac{\mathsf{P}_{\mathsf{out}}(\mathsf{r}, \mathsf{snr})}{\mathsf{snr}^{-\mathsf{d}(\mathsf{r})}} = \mathsf{a}(\mathsf{r})^{-\mathsf{d}(\mathsf{r})}. \tag{11}$$

The multiplexing gain definition coincides exactly with the original DMT formulation in [17], while the diversity gain differs from that in [17] in the fact that we use the outage probability instead of the BLER. However, for the fundamental DMA analysis addressed in this paper, both definitions become equivalent, as the outage probability provides the best achievable BLER.

Observe that definitions in (10) and (11) induce the following approximation of the high-SNR behavior of the outage probability:

$$\mathsf{P}_{\mathsf{out}}(\mathsf{r}, \mathsf{snr}) \sim \left(\mathsf{a}(\mathsf{r}) \cdot \mathsf{snr}\right)^{-\mathsf{d}(\mathsf{r})} \tag{12}$$

where "$\sim$" denotes asymptotic equivalence as $\mathsf{snr} \to \infty$, i.e., $f(\mathsf{snr}) \sim g(\mathsf{snr})$ if $\lim_{\mathsf{snr}\to\infty} f(\mathsf{snr})/g(\mathsf{snr}) = 1$.

Hence, the DMA gains of a particular system can be directly translated into a parameterized characterization of its associated outage probability performance as opposed to the DMT, which only provides the slope of the outage probability curve for a given multiplexing gain. This enables the direct comparison of different strategies with equal diversity gain under different channel models and CSI assumptions.

A similar high-SNR affine characterization under ergodic channel conditions is proposed in [29], being, thus, the ergodic capacity the performance measure of interest. The authors in [29] point out the importance of extending their results to the nonergodic case when some of the degrees of freedom of the channel are sacrificed to increase the diversity gain, i.e., in different operational points of the DMT curve, and this is exactly the contribution of this paper. Our approach is also similar to that in [18], where an affine approximation of the high-SNR outage capacity is derived. The analysis in [18] is, however, restricted to systems with fixed rate or, equivalently, no multiplexing gain.

[2]The array gain definition in (11) holds whenever the limit exist as happens with the MIMO point-to-point channel models addressed in this paper. If this is not the case, the implicit definition in (12) should be adopted instead. See, for instance, for the DMA analysis of the half-duplex static MIMO relay channel in [28].

III. DMA Analysis of MIMO Systems

In this section, we derive the DMA gains for the channel models introduced in Section II-A. For technical reasons, we restrict our attention to the case in which the transmit covariance matrix $\mathbf{Q}$ is a scaled identity matrix, i.e.

$$\mathbf{Q} = \frac{\mathsf{snr}}{n_\mathsf{T}}\mathbf{I}_{n_\mathsf{T}}. \tag{13}$$

Then, following the standard approach of generating an equivalent system model by using the singular-value decomposition of the channel matrix (see, e.g., [30, Sec. II]):

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^\dagger\mathbf{X} + \mathbf{W} \tag{14}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices containing the singular vectors of $\mathbf{H}$ and $\mathbf{\Lambda} = \mathrm{diag}(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_n})$, with $\lambda_1 \geq \cdots \geq \lambda_n \geq 0$ denoting the nonzero ordered eigenvalues of $\mathbf{H}\mathbf{H}^\dagger$. Since the distribution of $\mathbf{W}$ is unitarily invariant [31, Sec. 2.1.5], we can rewrite (14) without loss of generality as

$$\tilde{\mathbf{Y}} = \mathbf{\Lambda}\tilde{\mathbf{X}} + \tilde{\mathbf{W}} \tag{15}$$

where $\tilde{\mathbf{Y}} = \mathbf{U}^\dagger\mathbf{Y}$, $\tilde{\mathbf{X}} = \mathbf{V}^\dagger\mathbf{X}$, and $\tilde{\mathbf{W}} = \mathbf{U}^\dagger\mathbf{W}$. Finally, given the transmit covariance matrix in (13) and assuming that R satisfies (9), it holds that

$$\mathsf{P}_{\mathsf{out}}(\mathsf{r}, \mathsf{snr}) \sim \mathsf{Pr}\Big(\prod_{t=1}^{n}\big(1 + \frac{\mathsf{snr}}{n_\mathsf{T}}\lambda_t\big) < \mathsf{snr}^{\mathsf{r}}\Big). \tag{16}$$

From (12), it is clear that the DMA analysis under the channel models in Definitions 1, 2, and 3 results from obtaining the first-order series expansion of the corresponding outage probability in (16). It is well known, however, that these channel models have the same DMT [32] as the one for uncorrelated Rayleigh MIMO channels derived in [17, Th. 1]. Hence, we can already present the diversity gain $\mathsf{d}(\mathsf{r})$ as a function of the multiplexing gain r in the next lemma.

Lemma 1: The diversity gain $\mathsf{d}(\mathsf{r})$ in an $n_\mathsf{R} \times n_\mathsf{T}$ MIMO system with multiplexing gain $0 \leq \mathsf{r} \leq n$ and isotropic signaling as in (13) is given for the channel models in Definitions 1, 2, and 3 by

$$\mathsf{d}(\mathsf{r}) = G_\mathsf{d}(k) - G_\mathsf{r}(k)\mathsf{r} \tag{17}$$

where $k = \lfloor \mathsf{r} \rfloor = \{\max k \in \mathbb{N} | k \leq \mathsf{r}\}$, and

$$G_\mathsf{d}(k) = mn - k(k+1) \tag{18}$$
$$G_\mathsf{r}(k) = m + n - (2k+1) \tag{19}$$

with $n = \min(n_\mathsf{T}, n_\mathsf{R})$ and $m = \max(n_\mathsf{T}, n_\mathsf{R})$.

Proof: See [32, Th. 2, Th. 3, and Corollary 3] or Appendices A.i–A.iii.

Now, in order to complete the DMT framework in Lemma 1, we only need to obtain the array gain as done in the next theorem.

Theorem 1: The array gain $\mathsf{a}(\mathsf{r})$ in an $n_\mathsf{R} \times n_\mathsf{T}$ MIMO system with multiplexing gain $0 < \mathsf{r} < n$ and isotropic signaling as in (13) is given as follows.

i. For uncorrelated Rayleigh fading (see Definition 1):

$$\mathsf{a}_1(\mathsf{r}) = \left(K_1(k)|\mathbf{A}(k)|\Big(\frac{n_\mathsf{T}^{G_\mathsf{d}(k)}}{G_\mathsf{r}(k)}\Big)\right)^{-1/\mathsf{d}(\mathsf{r})} \tag{20}$$

where

$$K_1(k) = \frac{\prod_{t=1}^{k} t!}{\prod_{t=1}^{n-k}(k+t-1)!\prod_{t=1}^{n}(m-t)!} \tag{21}$$

matrix $\mathbf{A}(k)$ $\big((n-k-1) \times (n-k-1)\big)$ is defined as

$$[\mathbf{A}(k)]_{u,v} = \sum_{t=0}^{m-n}\binom{m-n}{t}\frac{(-1)^t}{(u+v+t)} \tag{22}$$

and the rest of parameters are as given in Lemma 1.

ii. For semicorrelated Rayleigh fading (see Definition 2):

$$\mathsf{a}_2(\mathsf{r}) = \left(K_2(k,\boldsymbol{\sigma})|\mathbf{A}_2(k,\boldsymbol{\sigma})||\mathbf{A}(k)|\Big(\frac{n_\mathsf{T}^{G_\mathsf{d}(k)}}{G_\mathsf{r}(k)}\Big)\right)^{-1/\mathsf{d}(\mathsf{r})} \tag{23}$$

where $\boldsymbol{\sigma}$ denote the ordered eigenvalues of the channel correlation matrix $\mathbf{\Sigma}$,

$$K_2(k,\boldsymbol{\sigma}) = \frac{(-1)^{k(n-k)}}{\prod_{t=1}^{n}\sigma_t^m(m-t)!}\prod_{i<j}^{n}\Big(\frac{\sigma_i\sigma_j}{\sigma_j - \sigma_i}\Big) \tag{24}$$

and matrix $\mathbf{A}_2(k,\boldsymbol{\sigma})$ $(n \times n)$ is defined in (25), shown at the bottom of the page.

iii. For uncorrelated Rician fading (see Definition 3):

$$\mathsf{a}_3(\mathsf{r}) = \left(K_3(k,\mathsf{K},\boldsymbol{\omega})|\mathbf{A}_3(k,\boldsymbol{\omega})||\mathbf{A}(k)|\Big(\frac{n_\mathsf{T}^{G_\mathsf{d}(k)}}{G_\mathsf{r}(k)}\Big)\right)^{-1/\mathsf{d}(\mathsf{r})} \tag{26}$$

where K is the Rician factor and $\boldsymbol{\omega}$ denote the ordered eigenvalues of the noncentrality matrix $\mathbf{\Omega}$ defined in (5):

$$K_3(k,\mathsf{K},\boldsymbol{\omega}) = (-1)^{k(n-k)}(\mathsf{K}+1)^{G_\mathsf{d}(k)}e^{-mn\mathsf{K}}\prod_{i<j}^{n}\Big(\frac{1}{\omega_j - \omega_i}\Big) \tag{27}$$

$$[\mathbf{A}_2(k,\boldsymbol{\sigma})]_{u,v} = \begin{cases} \frac{1}{(n-k-v-1)!}\big(\frac{-1}{\sigma_u}\big)^{(n-k)-v-1}\ln(\sigma_u) & 1 \leq v \leq \min(n-k-1,k) \\ (v-(n-k))!\big(\frac{1}{\sigma_u}\big)^{(n-k)-v-1} & n-k \leq v \leq k \\ \frac{1}{(v-k-1)!}\big(\frac{-1}{\sigma_u}\big)^{v-k-1} & k < v \leq n. \end{cases} \tag{25}$$

and matrix $\mathbf{A}_3(k,\boldsymbol{\omega})$ $(n \times n)$ is defined as

$$[\mathbf{A}_3(k,\boldsymbol{\omega})]_{u,v} = \begin{cases} \frac{v!}{(m-k)!(n-k)!} \mathcal{F}_v(k,\omega_u) & 1 \le v \le k \\ \frac{1}{(v-k-1)!(m-n+v-k-1)!} \omega_u^{v-k-1} & k < v \le n \end{cases} \quad (28)$$

where $\mathcal{F}_v(k,\omega_u) = \omega_u^{n-k} {}_2F_2(v+1,1;m-k+1,n-k+1;\omega_u)$ with ${}_2F_2(\cdot;\cdot)$ denoting the generalized hypergeometric function [33, eq. (9.14.1)].

Proof: See Appendices A.i–A.iii.

Remark 1.1: The determinant $|\mathbf{A}(k)|$ in (20), (23), and (26) can be evaluated in closed form using the multilinear property of determinants [34, Sec. 0.3.6], as shown in (29) at the bottom of the page, where the summation over $\boldsymbol{j} = (j_1, \ldots, j_{n-k-1})$ is for all $\boldsymbol{j} \in \{0, \ldots, m-n\}^{n-k-1}$ and last equation involves the evaluation of Cauchy's double alternant [35, eq. (2.7)].

Since the channel models considered in our analysis offer equal diversity gain but different array gain, the DMA analysis elucidates the performance gap between them. Choosing the uncorrelated Rayleigh fading channel as the reference channel, we define the asymptotic SNR gap as

$$\Delta_j(\mathsf{r}) = 10 \log_{10}\left(\frac{\mathsf{a}_j(\mathsf{r})}{\mathsf{a}_1(\mathsf{r})}\right) \quad \text{dB} \quad (30)$$

where $\mathsf{a}_j(\mathsf{r})$ denotes the array gain under the channel model in Definition j and $j \in \{2,3\}$. Observe that a positive $\Delta_j(\mathsf{r})$ indicates that the uncorrelated Rayleigh outage probability is outperformed, whereas a negative $\Delta_j(\mathsf{r})$ implies the opposite situation. In other words, $\Delta_j(\mathsf{r})$ indicates in how many dB we have to increase (if $\Delta_j(\mathsf{r}) < 0$) or reduce (if $\Delta_j(\mathsf{r}) > 0$) the nominal SNR to achieve the same outage probability as in the uncorrelated Rayleigh case.

Corollary 1.1: The asymptotic SNR gap $\Delta_j(\mathsf{r})$ at a multiplexing gain point $0 < \mathsf{r} < n$ with respect to the uncorrelated Rayleigh channel (see Definition 1) is given as follows.

i. For semicorrelated Rayleigh fading (see Definition 2):

$$\Delta_2(\mathsf{r}) = \frac{10}{\mathsf{d}(\mathsf{r})}\log_{10}(K_1(k)) - \frac{10}{\mathsf{d}(\mathsf{r})}\log_{10}(K_2(k,\boldsymbol{\sigma})|\mathbf{A}_2(k,\boldsymbol{\sigma})|) \quad \text{dB}. \quad (31)$$

ii. For uncorrelated Rician fading (see Definition 3):

$$\Delta_3(\mathsf{r}) = \frac{10}{\mathsf{d}(\mathsf{r})}\log_{10}(K_1(k)) - \frac{10}{\mathsf{d}(\mathsf{r})}\log_{10}(K_3(k,\mathsf{K},\boldsymbol{\omega})|\mathbf{A}_3(k,\boldsymbol{\omega})|) \quad \text{dB}. \quad (32)$$

For illustrative purposes, we show in Fig. 1 the numerical outage probability and the high-SNR outage probability characterization derived from the DMA analysis for the three addressed channel models. The numerical results have been obtained combining conventional Monte Carlo simulations with numerical integration techniques. Under the semicorrelated Rayleigh channel model, the correlation matrices are $[\boldsymbol{\Sigma}]_{i,j} = 0.7^{|i-j|}$ for $i,j = 1,\ldots,\min(n_\mathsf{T}, n_\mathsf{R})$, whereas, for the uncorrelated Rician channel model, $\mathsf{K} = 0$ dB and the LOS matrices have been randomly generated for each antenna setup. Since the target data rate is $\mathsf{R} = \mathsf{r}\log\mathsf{snr}$, the numerical outage probability is not representative for low SNR values. The remaining part of the outage probability curve is, as expected, well approximated by the DMA analysis. We emphasize that the traditional DMT (Lemma 1) provides the slope of the curves but not the horizontal shift, which is precisely the contribution of this paper (Theorem 1).

In addition, we can observe in Fig. 1 that the SNR gap for a fixed performance under different channel models is approximately constant in the range of outage probabilities of interest. Hence, the asymptotic SNR gap given in Corollary 1.1 provides a good prediction on the performance degradation/gain due to the channel model.

IV. DMA Analysis of MIMO Systems With Asymptotically Fixed Rate

The array gain expressions in Theorem 1 only hold for the case in which the data rate scales logarithmically with snr, i.e., when R satisfies (9) with $0 < \mathsf{r} < n$. When $\mathsf{r} \ge n$, the data rate scales with snr faster than the ergodic capacity of the channel and, hence, $\mathsf{d}(\mathsf{r}) = 0$ [17], i.e.

$$\lim_{\mathsf{snr}\to\infty} \frac{\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr})}{\log\mathsf{snr}} = 0. \quad (33)$$

In this section, we address the case in which the rate $\mathsf{R}(\mathsf{snr})$ is fixed or tends to a fixed value

$$\lim_{\mathsf{snr}\to\infty} \mathsf{R}(\mathsf{snr}) = \bar{\mathsf{R}} \quad (34)$$

$$|\mathbf{A}(k)| = \sum_{\boldsymbol{j}}\sum_{\boldsymbol{\mu}} \mathrm{sgn}(\boldsymbol{\mu}) \prod_{t=1}^{n-k-1} \binom{m-n}{j_t} \frac{(-1)^{j_t}}{(\mu_t + t + j_t)} = \sum_{\boldsymbol{j}} \left(\prod_{t=1}^{n-k-1}\binom{m-n}{j_t}(-1)^{j_t}\right) \sum_{\boldsymbol{\mu}} \mathrm{sgn}(\boldsymbol{\mu}) \prod_{t=1}^{n-k-1} \frac{1}{(\mu_t + t + j_t)}$$
$$= \sum_{\boldsymbol{j}} \frac{(-1)^{\sum_{u=1}^{n-k-1} j_u} \left((m-n)!\right)^{n-k-1} \prod_{u<v}^{n-k-1}(u-v)(u-v+j_u-j_v)}{\prod_{u=1}^{n-k-1}(m-n-j_u)! j_u! \prod_{v=1}^{n-k-1}(u+v+j_v)} \quad (29)$$

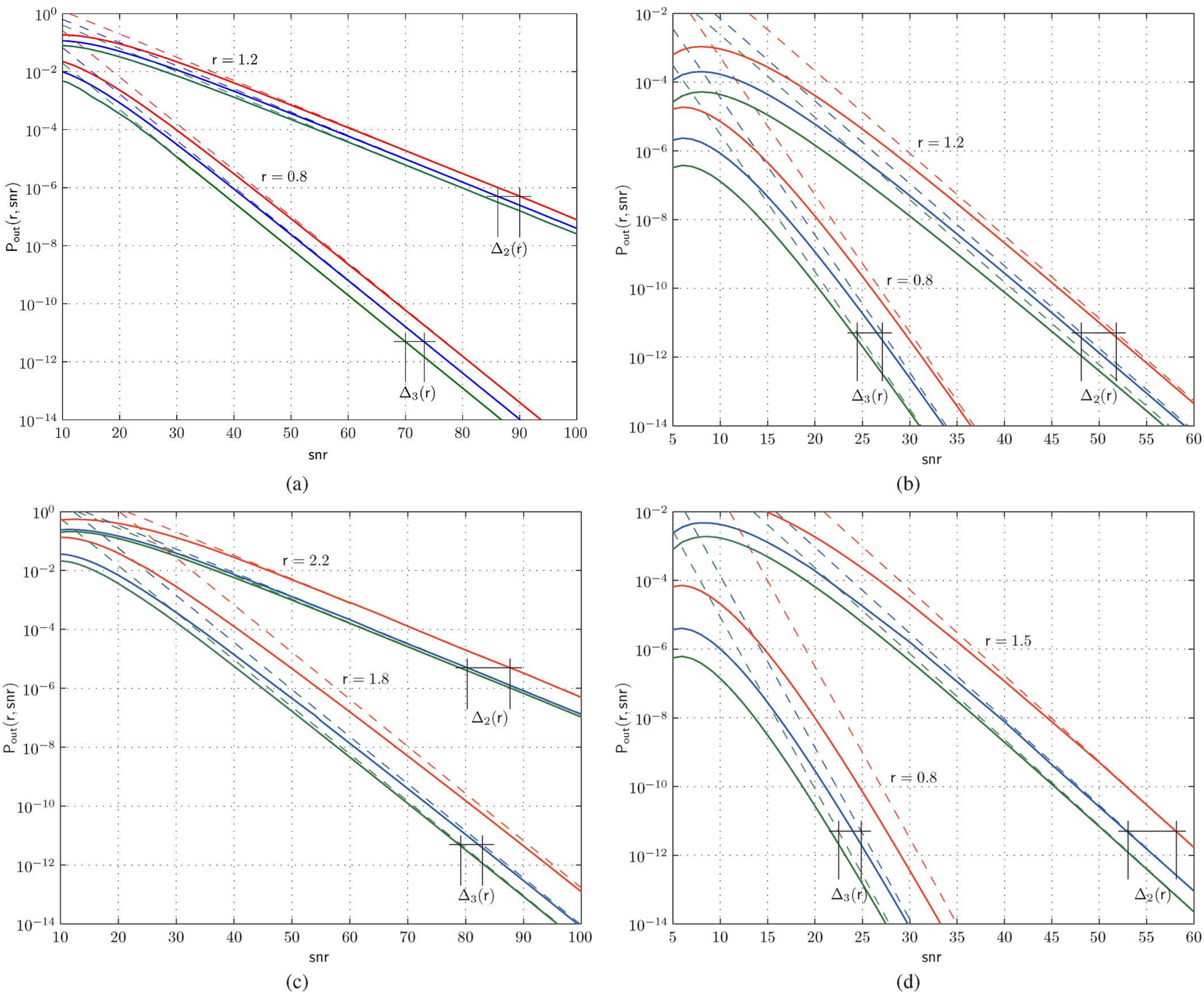

Fig. 1. Numerical outage probability (solid) and DMA analysis (dashed) under uncorrelated Rayleigh fading (blue), semicorrelated Rayleigh fading (red), and uncorrelated Rician fading (green). (a) $n_\mathsf{T} = 2$ and $n_\mathsf{R} = 2$. (b) $n_\mathsf{T} = 2$ and $n_\mathsf{R} = 4$. (c) $n_\mathsf{T} = 3$ and $n_\mathsf{R} = 3$. (d) $n_\mathsf{T} = 3$ and $n_\mathsf{R} = 3$.

and, hence, $\mathsf{r} = 0$. This case is important to analyze those schemes whose rate adaptation policy saturates at certain rate $\bar{\mathsf{R}}$ or do not modify the transmission rate at all. The DMA analysis under this assumption follows from using the diversity gain $\mathsf{d}(0) = n_\mathsf{T} n_\mathsf{R}$ predicted by Lemma 1 and the array gain $\mathsf{a}(\bar{\mathsf{R}})$ presented in the next theorem.

Theorem 2: The array gain $\mathsf{a}(\bar{\mathsf{R}})$ in an $n_\mathsf{R} \times n_\mathsf{T}$ MIMO system when the data rate satisfies (34) and with isotropic signaling as in (13) is given as follows.

i. For uncorrelated Rayleigh fading (see Definition 1):

$$\mathsf{a}_1(\bar{\mathsf{R}}) = \frac{1}{n_\mathsf{T}} \Big(K_{m,n} \sum_{\boldsymbol{\mu}} \operatorname{sgn}(\boldsymbol{\mu}) A(\boldsymbol{\mu}; \bar{\mathsf{R}}) \Big)^{-1/mn} \tag{35}$$

where $K_{m,n}$ is given in (44), the summation over $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)$ is for all permutations of integers $(1, \ldots, n)$, $\operatorname{sgn}(\cdot)$ denotes the sign of the permutation, and $A(\boldsymbol{\mu}; \bar{\mathsf{R}})$ is defined as

$$A(\boldsymbol{\mu}; \bar{\mathsf{R}}) = \int_1^{2^{\bar{\mathsf{R}}}} \int_1^{\left(\frac{2^{\bar{\mathsf{R}}}}{x_1}\right)} \cdots \int_1^{\left(\frac{2^{\bar{\mathsf{R}}}}{\prod_{t=1}^{n-1} x_t}\right)} \prod_{t=1}^{n} x_t^{\mu_t + t - 2} (x_t - 1)^{m-n} dx_n \cdots dx_1. \tag{36}$$

ii. For semicorrelated Rayleigh fading (see Definition 2):

$$\mathsf{a}_2(\bar{\mathsf{R}}) = |\boldsymbol{\Sigma}|^{1/n} \mathsf{a}_1(\bar{\mathsf{R}}). \tag{37}$$

iii. For uncorrelated Rician fading (see Definition 3):

$$\mathsf{a}_3(\bar{\mathsf{R}}) = \frac{e^{\mathsf{K}}}{(\mathsf{K}+1)} \mathsf{a}_1(\bar{\mathsf{R}}). \tag{38}$$

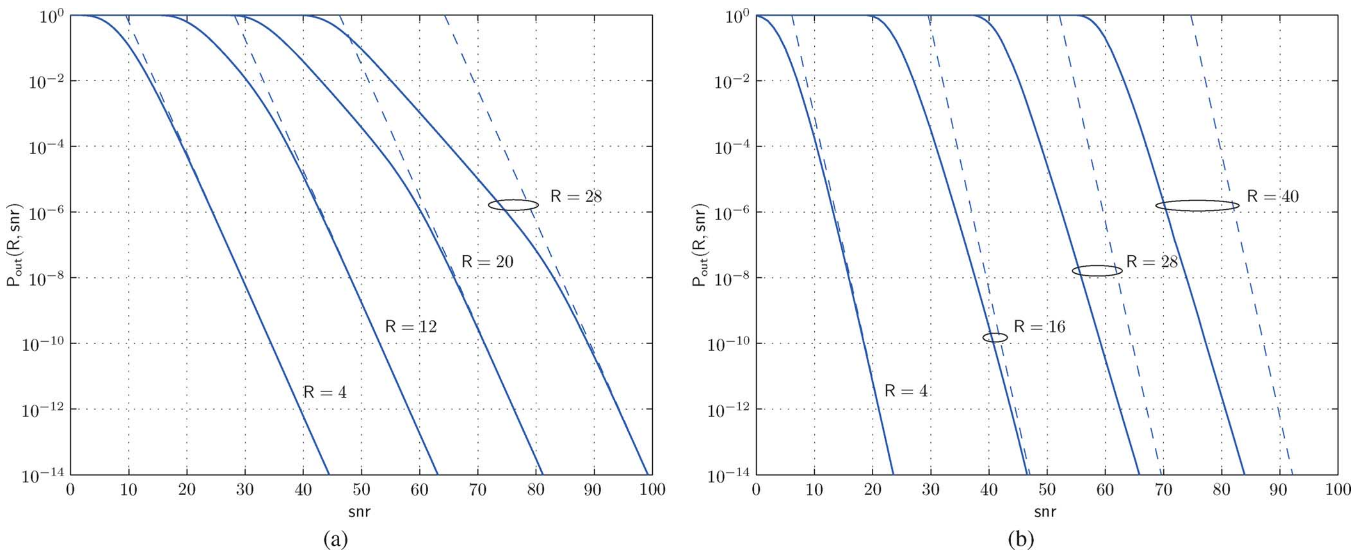

Fig. 2. Numerical outage probability (solid) and DMA analysis (dashed) under uncorrelated Rayleigh fading. (a) $n_\mathsf{T} = 2$ and $n_\mathsf{R} = 2$. (b) $n_\mathsf{T} = 2$ and $n_\mathsf{R} = 4$.

Proof: See Appendix B.

Observe that Theorem 2 implicitly gives the following asymptotic SNR gaps with respect to the uncorrelated Rayleigh channel:

$$\Delta_2 = -\frac{10}{n}\log_{10}|\mathbf{\Sigma}| \quad \text{dB} \tag{39}$$

$$\Delta_3 = 10\log_{10}\left(\frac{e^{\mathsf{K}}}{\mathsf{K}+1}\right) \quad \text{dB}. \tag{40}$$

The SNR gap for uncorrelated Rician channels coincides with that derived in [36, Th. 1], while the strictly negative nature of the SNR gap for semicorrelated Rayleigh channels recalls the known fact that the outage probability with isotropic inputs is increased by antenna correlation [37]. As in the ergodic case addressed in [29], this penalty can be arbitrarily large if one or some of the eigenvalues of the correlation matrix $\mathbf{\Sigma}$ are small.

Finally, it is worth remarking that assumption (34) does not include all rate adaptation strategies with $\mathsf{r} = 0$. It remains to consider the case in which the rate scales sublogarithmically with snr but does not saturate, i.e.,

$$\lim_{\mathsf{snr}\to\infty}\frac{\mathsf{R}(\mathsf{snr})}{\log\mathsf{snr}} = 0 \quad \text{and} \quad \lim_{\mathsf{snr}\to\infty}\mathsf{R}(\mathsf{snr}) = \infty. \tag{41}$$

This case, however, has less practical interest and can be easily obtained combining the methods used in the proofs of Theorems 1 and 2.

For illustrative purposes, we show in Fig. 2 the numerical outage probability and the high-SNR outage probability characterization derived from the DMA analysis under uncorrelated Rayleigh fading when the transmission rate R is fixed. Again, we assume isotropic inputs and obtain the numerical results using Monte Carlo simulations and numerical integration techniques. In Fig. 2, we see that the DMA analysis for fixed rate in Theorem 2 does not always provide an acceptable approximation, as happens with Theorem 1 when $\mathsf{R}(\mathsf{snr})$ scales with $\log\mathsf{snr}$. More exactly, the DMA characterization does not capture the outage probability behavior in the SNR region of interest for high R values. The reason is simple; when R is high, we need impractically large SNR values for the assumption $\mathsf{R}/\log\mathsf{snr} \to 0$ to hold. Furthermore, this effect is hardened when the number of receive and/or transmit antennas is increased, since the higher associated diversity gains result in even lower outage probabilities at moderate SNR values.

The previous observations suggest that a better approximation can be obtained by removing the assumption that $\mathsf{R}/\log\mathsf{snr} \to 0$. Following this intuition, we can apply Theorem 1 with $\mathsf{r} = \mathsf{R}/\log\mathsf{snr}$ and obtain a different affine characterization of the outage probability for each $k = \lfloor \mathsf{R}/\log\mathsf{snr}\rfloor \in \{0,\ldots,n-1\}$. Then, taking the minimum among these individual DMA curves, the outage probability with fixed R is approximated (for instance, for the uncorrelated Rayleigh channel) as[3]

$$\mathsf{P}_{\mathsf{out}}(\mathsf{R},\mathsf{snr}) \approx \min_k\left(K_1(k)|\mathbf{A}(k)|e^{G_{\mathsf{r}}(k)\mathsf{R}}\left(\frac{n_\mathsf{T}^{G_\mathsf{d}(k)}}{G_\mathsf{r}(k)}\right)\right)\mathsf{snr}^{-G_\mathsf{d}(k)}. \tag{42}$$

The resulting piecewise linear approximation is heuristic due to its asymptotic nature and, hence, it becomes more accurate for large R and when snr increases. This can be observed in Fig. 3, where we plot the numerical outage probability and the proposed asymptotic DMA approximation. The outage probability behavior is perfectly captured for small-size MIMO systems. However, the effect of using an asymptotic formulation for the description of the finite rate case at finite SNR values is again hardened as the number of antennas increases.

V. Conclusion

Zheng and Tse 2003 paper was the first one to reveal and quantify the fundamental interconnection present in MIMO channels between the multiplexing gain, associated to rate, and the diversity gain, related to the slope of the error rate. This characterization is, however, difficult to be translated into practical performance indicators like block error rate without the addition of a third parameter: the array gain, which provides the

[3]This approach is similar to that in [24] with the advantage that it captures the effect of the channel model.

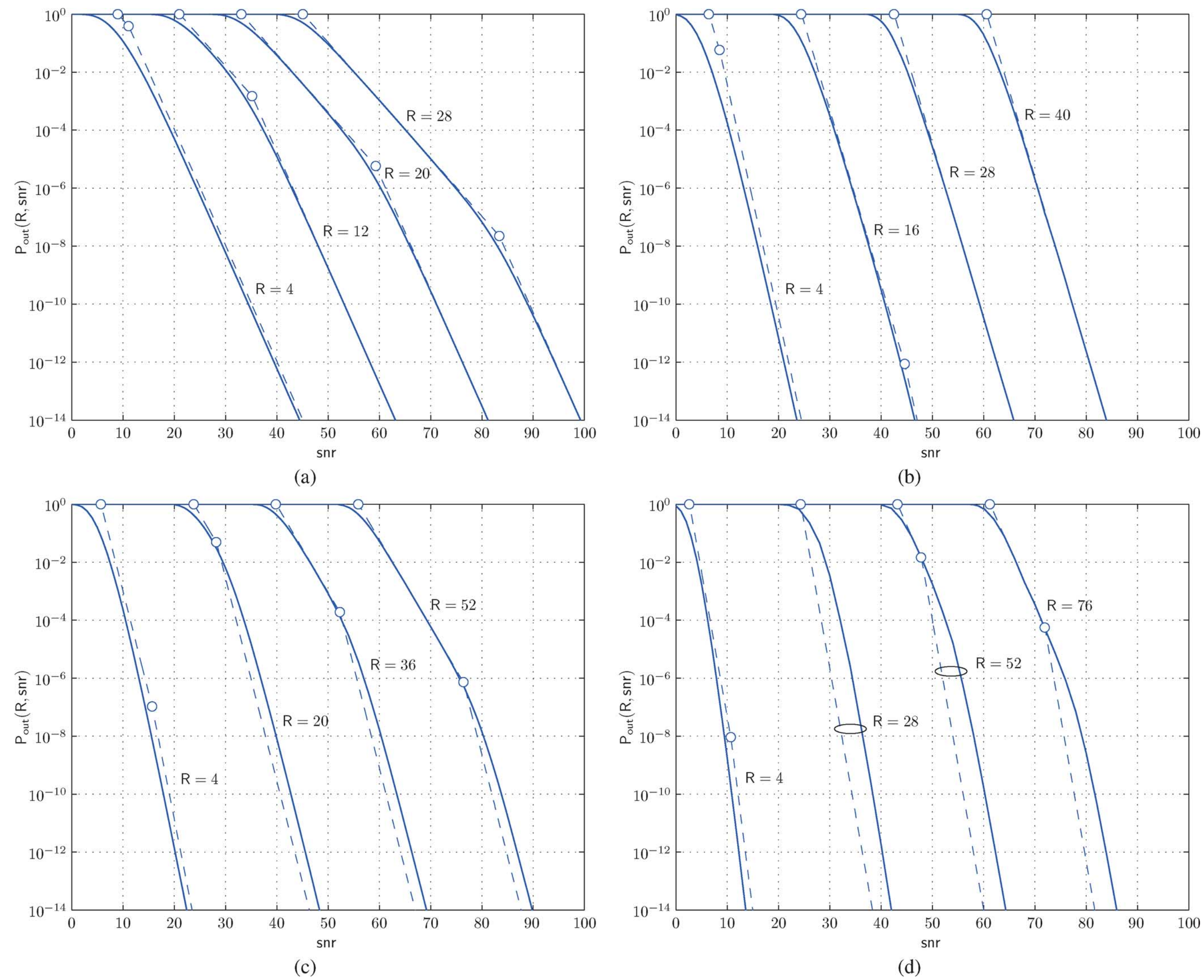

Fig. 3. Numerical outage probability (solid) and heuristic DMA approximation (dashed) under uncorrelated Rayleigh fading. (a) $n_{\mathsf{T}} = 2$ and $n_{\mathsf{R}} = 2$. (b)$n_{\mathsf{T}} = 2$ and $n_{\mathsf{R}} = 4$. (c) $n_{\mathsf{T}} = 3$ and $n_{\mathsf{R}} = 3$. (d) $n_{\mathsf{T}} = 4$ and $n_{\mathsf{R}} = 4$.

shift to the diversity gain slope resulting in an asymptotic affine characterization of the error curve in the logarithmic domain. This paper introduces the DMA analysis under uncorrelated Rayleigh, semicorrelated Rayleigh, and uncorrelated Rician MIMO channels, opening the door for a more illustrative performance evaluation of MIMO schemes.

Appendix A
Proof of Theorem 1

A.i. Proof of Theorem 1.i (Uncorrelated Rayleigh):

Under uncorrelated Rayleigh fading, either $\mathbf{H}\mathbf{H}^\dagger$ or $\mathbf{H}^\dagger\mathbf{H}$ is uncorrelated central Wishart distributed [25, Sec. II]. Since the nonzero ordered eigenvalues $\lambda_1 \geq \cdots \geq \lambda_n > 0$ of $\mathbf{H}\mathbf{H}^\dagger$ and $\mathbf{H}^\dagger\mathbf{H}$ coincide, the joint probability density function (pdf) of $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)$ is given by [38, eq. (95)]

$$f_{\boldsymbol{\lambda}}(\boldsymbol{\lambda}) = K_{m,n} \prod_{i<j}^{n} (\lambda_j - \lambda_i)^2 \prod_{t=1}^{n} e^{-\lambda_t} \lambda_t^{m-n} \tag{43}$$

where the normalization constant $K_{m,n}$ is

$$K_{m,n} = \prod_{t=1}^{n} \frac{1}{(n-t)!(m-t)!}. \tag{44}$$

Let us now introduce the ordered variables $\alpha_1 \geq \cdots \geq \alpha_n > 1$ with

$$\alpha_t = 1 + \frac{\mathsf{snr}}{n_{\mathsf{T}}} \lambda_t \qquad t = 1, \ldots, n \tag{45}$$

so that the outage probability in (16) can be rewritten in terms of $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)$ as

$$\mathsf{P}_{\text{out}}(\mathsf{r}, \mathsf{snr}) \sim \Pr\Big(\prod_{t=1}^{n} \alpha_t < \mathsf{snr}^{\mathsf{r}}\Big) = \int_{\mathcal{A}(\mathsf{r},\mathsf{snr})} f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) d\boldsymbol{\alpha} \tag{46}$$

where $\mathcal{A}(\mathsf{r},\mathsf{snr}) = \{\boldsymbol{\alpha} \in \mathbb{R}^n | \prod_{t=1}^n \alpha_t < \mathsf{snr}^{\mathsf{r}}, \alpha_1 \geq \cdots \geq \alpha_n \geq 1\}$ for $0 < \mathsf{r} < n$ and $f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ denotes the joint pdf of $\boldsymbol{\alpha}$ given by

$$f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) = K_{m,n}\left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{mn} e^{n(\frac{n_{\mathsf{T}}}{\mathsf{snr}})} \prod_{i<j}^{n}(\alpha_j-\alpha_i)^2 \prod_{t=1}^{n} e^{-(\frac{n_{\mathsf{T}}}{\mathsf{snr}})\alpha_t}(\alpha_t-1)^{m-n}. \quad (47)$$

In order to obtain the asymptotic characterization in (12), it is important to observe first that not all $\boldsymbol{\alpha} \in \mathcal{A}(\mathsf{r},\mathsf{snr})$ contribute to the high-SNR behavior of $\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr})$ in (46). This follows from using Laplace's principle for multidimensional integrals [39, Ch. 5], which states that the term with highest SNR exponent comes from integrating $f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ in a neighborhood of points $\mathcal{A}^\star(\mathsf{r},\mathsf{snr}) \subseteq \mathcal{A}(\mathsf{r},\mathsf{snr})$, where all $\boldsymbol{\alpha}^\star \in \mathcal{A}^\star(\mathsf{r},\mathsf{snr})$ satisfy

$$\boldsymbol{\alpha}^\star = \arg\min_{\boldsymbol{\alpha}\in\mathcal{A}(\mathsf{r},\mathsf{snr})} \sum_{i=1}^{n} \alpha_i \quad (48)$$

or, equivalently (see the proof of [17, Th. 4])

$$\log_{\mathsf{snr}}(\alpha_t^\star) = \begin{cases} 1 & 1 \leq t \leq k \\ \mathsf{r}-k & t = k+1 \\ 0 & k+1 < t \leq n \end{cases} \quad (49)$$

where $k = \lfloor \mathsf{r} \rfloor$. Observe now that, when $\boldsymbol{\alpha} \in \mathcal{A}^\star(\mathsf{r},\mathsf{snr})$, the following asymptotic equivalences hold:

$$\prod_{i=1}^{k}\prod_{j=k+1}^{n}(\alpha_j-\alpha_i)^2 \sim \prod_{t=1}^{k}\alpha_t^{2(n-k)} \quad (50)$$

$$\prod_{t=1}^{k}(\alpha_t-1)^{m-n} \sim \prod_{t=1}^{k}\alpha_t^{m-n} \quad (51)$$

$$e^{n(\frac{n_{\mathsf{T}}}{\mathsf{snr}})} \sim e^{-(\frac{n_{\mathsf{T}}}{\mathsf{snr}})\alpha_{k+1}} \sim \cdots \sim e^{-(\frac{n_{\mathsf{T}}}{\mathsf{snr}})\alpha_n} \sim 1 \quad (52)$$

since $\{\alpha_t\}_{t=1}^k$ are $O(\mathsf{snr})$, whereas $\{\alpha_t\}_{t=k+1}^n$ are $o(\mathsf{snr})$.[4]

Taking into account these asymptotics and noting that

$$\prod_{i<j}^{n}(\alpha_j-\alpha_i) = \prod_{i=1}^{k}\prod_{j=k+1}^{n}(\alpha_j-\alpha_i)\prod_{i<j}^{k}(\alpha_j-\alpha_i)\prod_{k<i<j}^{n}(\alpha_j-\alpha_i) \quad (53)$$

the high-SNR behavior of $\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr})$ in (46) is given by

$$\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr}) \sim K_{m,n}\left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{mn}\int_{\mathcal{A}^\star(\mathsf{r},\mathsf{snr})}\prod_{t=1}^{k}e^{-(\frac{n_{\mathsf{T}}}{\mathsf{snr}})\alpha_t}\alpha_t^{\zeta(2k)} \prod_{i<j}^{k}(\alpha_j-\alpha_i)^2\prod_{k<i<j}^{n}(\alpha_j-\alpha_i)^2\prod_{t=k+1}^{n}(\alpha_t-1)^{m-n}d\boldsymbol{\alpha} \quad (54)$$

[4]We say that $f(\mathsf{snr}) = O(g(\mathsf{snr}))$ as $\mathsf{snr} \to \infty$ if there exist numbers a and A such that $|f(\mathsf{snr})| \leq A|g(\mathsf{snr})|$ for $a < \mathsf{snr} < \infty$ [40, eq. (1.2.7)] and we say that $f(\mathsf{snr}) = o(g(\mathsf{snr}))$ if $f(\mathsf{snr})/g(\mathsf{snr}) \to 0$ as $\mathsf{snr} \to \infty$ [40, eq. (1.3.1)].

where $\zeta(t) = m+n-t$. Let us define

$$\Lambda_{(k)}(\mathsf{r},\mathsf{snr}) = \int_{\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr})}\prod_{t=1}^{k}e^{-(\frac{n_{\mathsf{T}}}{\mathsf{snr}})\alpha_t}\alpha_t^{\zeta(2k)} \prod_{i<j}^{k}(\alpha_j-\alpha_i)^2\Lambda^{(k)}(\boldsymbol{\alpha}_k;\mathsf{r},\mathsf{snr})d\alpha_k\cdots d\alpha_1 \quad (55)$$

$$\Lambda^{(k)}(\boldsymbol{\alpha}_k;\mathsf{r},\mathsf{snr}) = \int_{\mathcal{A}^{(k)}(\boldsymbol{\alpha}_k,\mathsf{r},\mathsf{snr})}\prod_{k<i<j}^{n}(\alpha_j-\alpha_i)^2 \prod_{t=k+1}^{n}(\alpha_t-1)^{m-n}d\alpha_n\cdots d\alpha_{k+1} \quad (56)$$

where $\boldsymbol{\alpha}_k = (\alpha_1,\ldots,\alpha_k)$

$$\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr}) = \Big\{(\alpha_1,\ldots,\alpha_k) \in \mathbb{R}^k \mid \prod_{t=1}^{k}\alpha_t < \mathsf{snr}^{\mathsf{r}}, \alpha_1 \geq \cdots \geq \alpha_k \geq 1\Big\} \quad (57)$$

$$\mathcal{A}^{(k)}(\boldsymbol{\alpha}_k,\mathsf{r},\mathsf{snr}) = \Big\{(\alpha_{k+1},\ldots,\alpha_n) \in \mathbb{R}^{n-k} \mid \prod_{t=1}^{n}\alpha_t < \mathsf{snr}^{\mathsf{r}},\ \alpha_{k+1} \geq \cdots \geq \alpha_n \geq 1\Big\} \quad (58)$$

and (49) is satisfied. This guarantees that $\alpha_k \geq \alpha_{k+1}$ and, hence, integrating over $\mathcal{A}^{(k)}(\boldsymbol{\alpha}_k,\mathsf{r},\mathsf{snr})$ and subsequently over $\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr})$ is equivalent to integrating directly over $\mathcal{A}^\star(\mathsf{r},\mathsf{snr})$. We can now rewrite (54) as

$$\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr}) \sim K_{m,n}\left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{mn}\Lambda_{(k)}(\mathsf{r},\mathsf{snr}). \quad (59)$$

The integral $\Lambda^{(k)}(\boldsymbol{\alpha}_k;\mathsf{r},\mathsf{snr})$ in (56) is shown in Appendix A.i.1 to satisfy

$$\Lambda^{(k)}(\boldsymbol{\alpha}_k;\mathsf{r},\mathsf{snr}) \sim \frac{1}{\zeta(2k+1)}|\mathbf{A}(k)|\left(\frac{\mathsf{snr}^{\mathsf{r}}}{\prod_{t=1}^{k}\alpha_t}\right)^{\zeta(2k+1)} \quad (60)$$

where matrix $\mathbf{A}(k)$ is defined in (74). Furthermore, the asymptotic behavior of $\Lambda_{(k)}(\mathsf{r},\mathsf{snr})$, when substituting (60) back in (55), is shown in Appendix A.i.2 to be given by

$$\Lambda_{(k)}(\mathsf{r},\mathsf{snr}) \sim |\mathbf{A}(k)|\left(\prod_{t=1}^{k}(t-1)!t!\right)\left(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\right)\left(\frac{\mathsf{snr}}{n_{\mathsf{T}}}\right)^{k(k+1)}. \quad (61)$$

Finally, combining (61) with (59), the array gain $\mathsf{a}_1(\mathsf{r})$ provided in Theorem 1 for the uncorrelated Rayleigh channel follows. □

A.i.1) Integral $\Lambda^{(k)}(\boldsymbol{\alpha}_k;\mathsf{r},\mathsf{snr})$: The objective of this appendix is to derive the asymptotic behavior of the integral $\Lambda^{(k)}(\boldsymbol{\alpha}_k;\mathsf{r},\mathsf{snr})$ defined in (56).

Since $\prod_{k<i<j}^n(\alpha_j-\alpha_i)^2 = |\mathbf{V}(\alpha_{k+1},\ldots,\alpha_n)|^2$ with $|\mathbf{V}(\cdot)|$ denoting the Vandermonde determinant of order $n-k$ [41, eq. (6.1.33)], it follows that

$$\prod_{k<i<j}^{n}(\alpha_j-\alpha_i)^2 = \sum_{\boldsymbol{\mu},\boldsymbol{\nu}}\mathrm{sgn}(\boldsymbol{\mu})\mathrm{sgn}(\boldsymbol{\nu})\prod_{t=1}^{n-k}\alpha_{t+k}^{\mu_t+\nu_t-2} \quad (62)$$

where the summation over $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_{n-k})$ and $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_{n-k})$ is for all permutations of integers $(1, \ldots, n-k)$. Substituting back in (56) and exploiting the symmetry of the integrand (see details, for instance, in the proof of [25, Th. 1]), it holds that

$$\Lambda^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr}) = \frac{1}{(n-k)!} \int_{\tilde{\mathcal{A}}^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr})} \sum_{\boldsymbol{\mu}, \boldsymbol{\nu}} \operatorname{sgn}(\boldsymbol{\mu}) \operatorname{sgn}(\boldsymbol{\nu}) \prod_{t=1}^{n-k} \alpha_{t+k}^{\mu_t + \nu_t - 2} (\alpha_{t+k} - 1)^{m-n} d\alpha_n \cdots d\alpha_{k+1} \quad (63)$$

where $\tilde{\mathcal{A}}^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr})$ denotes the corresponding unordered domain, i.e., $\tilde{\mathcal{A}}^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr}) = \{(\alpha_{k+1}, \ldots, \alpha_n) \in \mathbb{R}^{n-k} | \prod_{t=1}^{n} \alpha_t < \mathsf{snr}^{\mathsf{r}}, \alpha_{k+1}, \ldots, \alpha_n \geq 1\}$.

Again, due to the symmetry of the integrand in (63), we can replace the integration variables $(\alpha_{k+1}, \ldots, \alpha_n)$ with any permutation of them. For instance, we can choose $\alpha_{k+t} = \alpha_{k+\nu_t}$, and obtain

$$\Lambda^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr}) = \sum_{\boldsymbol{\mu}} \operatorname{sgn}(\boldsymbol{\mu}) \int_{\tilde{\mathcal{A}}^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr})} \prod_{t=1}^{n-k} \alpha_{t+k}^{\mu_t + t - 2} (\alpha_{t+k} - 1)^{m-n} d\alpha_n \cdots d\alpha_{k+1}. \quad (64)$$

For simplicity of notation, let us now introduce

$$\Lambda_{n-k}(\xi(\boldsymbol{\mu}); x) = \int_1^x \int_1^{\left(\frac{x}{x_1}\right)} \cdots \int_1^{\left(\frac{x}{\prod_{t=1}^{n-k-1} x_t}\right)} \prod_{t=1}^{n-k} x_t^{\xi(\mu_t) - 2} (x_t - 1)^{m-n} dx_{n-k} \cdots dx_1 \quad (65)$$

so that

$$\Lambda^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr}) = \sum_{\boldsymbol{\mu}} \operatorname{sgn}(\boldsymbol{\mu}) \Lambda^{(k)}(\xi(\boldsymbol{\mu}); x) \quad (66)$$

for $x = \left(\frac{\mathsf{snr}^{\mathsf{r}}}{\prod_{t=1}^{k} \alpha_t}\right)$ and $\xi(\mu_t) = \mu_t + t$ for $t = 1, \ldots, n-k$. Then, using Newton's binomial [42, eq. (3.1.1)] to expand $(x_t - 1)^{m-n}$ as

$$(x_t - 1)^{m-n} = \sum_{i=0}^{m-n} \binom{m-n}{i} (-1)^i x_t^{m-n-i} \quad (67)$$

and integrating with respect to x_{n-k}, it follows that

$$\Lambda_{n-k}(\xi(\boldsymbol{\mu}); x) = \sum_{i=0}^{m-n} \binom{m-n}{i} (-1)^i \frac{1}{d(\xi(\mu_{n-k}) - i)} \Big(x^{d(\xi(\mu_{n-k}) - i)} \Lambda_{n-k-1}\big(\xi(\mu_1) - d(\xi(\mu_{n-k}) - i), \ldots, \xi(\mu_{n-k-1}) - d(\xi(\mu_{n-k}) - i); x\big) \quad (68)$$
$$- \Lambda_{n-k-1}\big(\xi(\mu_1), \ldots, \xi(\mu_{n-k-1}); x\big)\Big) \quad (69)$$

with $d(t) = m - n + t - 1$. Observe that exponent of $x^{d(\xi(\mu_{n-k}) - i)}$ is maximum when $i = 0$ and $\mu_{n-k} = n - k$, leading to $d(2(n-k))$. If we keep integrating the term in (68) and we evaluate in the upper limit, the exponent only gets reduced, since $d(\xi(\mu_t) - i) < d(2(n-k))$ for any $t < n-k$ and any $0 < i \leq m - n$. Following the same reasoning, the exponent of the integral in (69) is also strictly smaller than $d(2(n-k))$. Thus, for $\boldsymbol{\mu} = (\tau_1, \ldots, \tau_{n-k-1}, n-k)$ where $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_{n-k-1})$ is any permutation of integers $(1, \ldots, n-k-1)$, we have that

$$\Lambda_{n-k}(\xi(\boldsymbol{\mu}); x) \sim K(\xi(\boldsymbol{\tau})) \frac{x^{d(2(n-k))}}{d(2(n-k))} \quad (70)$$

where the constant $K(\cdot)$ follows from evaluating the $n-k-1$ integrals of (68) in the lower limit and is given by

$$K(\xi(\boldsymbol{\tau})) = \prod_{t=1}^{n-k-1} \sum_{i=0}^{m-n} \binom{m-n}{i} \frac{(-1)^i}{(2(n-k) - \xi(\tau_t) + i)}. \quad (71)$$

Now, recovering the equality in (66), it finally holds that

$$\Lambda^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr}) \sim \frac{1}{\zeta(2k+1)} \Big(\sum_{\boldsymbol{\tau}} \operatorname{sgn}(\boldsymbol{\tau}) K(\xi(\boldsymbol{\tau})) \Big) \Big(\frac{\mathsf{snr}^{\mathsf{r}}}{\prod_{t=1}^{k} \alpha_t} \Big)^{\zeta(2k+1)} \quad (72)$$

where $\zeta(t) = m + n - t$ and $\sum_{\boldsymbol{\tau}} \operatorname{sgn}(\boldsymbol{\tau}) K(\xi(\boldsymbol{\tau}))$ has the form of a determinant

$$|\mathbf{A}(k)| = \sum_{\boldsymbol{\tau}} \operatorname{sgn}(\boldsymbol{\tau}) \prod_{t=1}^{n-k-1} \sum_{i=0}^{m-n} \binom{m-n}{i} \frac{(-1)^i}{(\tau_t + t + i)} \quad (73)$$

and matrix $\mathbf{A}(k)$ $((n-k-1) \times (n-k-1))$ is, consequently, defined as

$$[\mathbf{A}(k)]_{u,v} = \sum_{i=0}^{m-n} \binom{m-n}{i} \frac{(-1)^i}{(u+v+i)}. \quad (74)$$

A.i.2) Integral $\Lambda_{(k)}(\mathsf{r}, \mathsf{snr})$: The objective of this appendix is to derive the asymptotic behavior of

$$\Lambda_{(k)}(\mathsf{r}, \mathsf{snr}) \sim |\mathbf{A}(k)| \Big(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)} \Big) \int_{\mathcal{A}_{(k)}(\mathsf{r}, \mathsf{snr})} \prod_{t=1}^{k} e^{-(\frac{n_{\mathrm{T}}}{\mathsf{snr}})\alpha_t} \alpha_t \prod_{i<j}^{k} (\alpha_j - \alpha_i)^2 d\alpha_k \cdots d\alpha_1 \quad (75)$$

where $\mathcal{A}_{(k)}(\mathsf{r}, \mathsf{snr})$ is defined in (57), $\zeta(t) = m + n - t$, and matrix $\mathbf{A}(k)$ is defined in (74).

First, we expand $\prod_{i<j}^{k} (\alpha_j - \alpha_i)^2$ as in Appendix A.i.1

$$\prod_{i<j}^{k} (\alpha_j - \alpha_i)^2 = \sum_{\boldsymbol{\mu}, \boldsymbol{\nu}} \operatorname{sgn}(\boldsymbol{\mu}) \operatorname{sgn}(\boldsymbol{\nu}) \prod_{t=1}^{k} \alpha_t^{\mu_t + \nu_t - 2} \quad (76)$$

where the summation over $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_k)$ and $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_k)$ is for all permutations of integers $(1, \ldots, k)$. Substituting (76) back in (75) and exploiting the symmetry of the integrand, we have that

$$A_{(k)}(\mathsf{r}, \mathsf{snr}) \sim \frac{1}{k!} |\mathbf{A}(k)| \Big(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)} \Big) \sum_{\boldsymbol{\mu}, \boldsymbol{\nu}} \operatorname{sgn}(\boldsymbol{\mu}) \operatorname{sgn}(\boldsymbol{\nu}) \int_1^{\mathsf{snr}^{\mathsf{r}}} \cdots \int_1^{\left(\frac{\mathsf{snr}^{\mathsf{r}}}{\prod_{u=1}^{k-1} \alpha_u}\right)} \prod_{t=1}^{k} e^{-(\frac{n_{\mathrm{T}}}{\mathsf{snr}})\alpha_t} \alpha_t^{\mu_t + \nu_t - 1} d\boldsymbol{\alpha}_k. \quad (77)$$

Each of the individual integrals in (77) satisfies

$$\int_1^{\left(\frac{\mathsf{snr}^{\mathsf{r}}}{\prod_{u=1}^{t-1}\alpha_u}\right)} e^{-\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\alpha_t}\alpha_t^{\xi_t-1}d\alpha_t = \left(\frac{\mathsf{snr}}{n_\mathsf{T}}\right)^{\xi_t} \left(\gamma\Big(\xi_t,\Big(\frac{n_\mathsf{T}}{\mathsf{snr}}\Big)\Big(\frac{\mathsf{snr}^{\mathsf{r}}}{\prod_{u=1}^{t-1}\alpha_u}\Big)\Big) - \gamma\Big(\xi_t,\Big(\frac{n_\mathsf{T}}{\mathsf{snr}}\Big)\Big)\right) \sim \left(\frac{\mathsf{snr}}{n_\mathsf{T}}\right)^{\xi_t}\Gamma(\xi_t) \quad (78)$$

where $\Gamma(\cdot)$ denotes the gamma function [42, eq. (6.1.1], $\gamma(\cdot,\cdot)$ denotes the lower incomplete gamma function [42, eq. (6.5.2)], and last equation comes from the fact that $\gamma(a,x) \sim \Gamma(a)$ as $x \to \infty$ and $\gamma(a,x) \sim x^a/a$ as $x \to 0$ [42, Sec. 6.5]. Accordingly, it holds that

$$\Lambda_{(k)}(\mathsf{r},\mathsf{snr}) \sim \frac{1}{k!}|\mathbf{A}(k)|\left(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\right) \left(\sum_{\boldsymbol{\mu},\boldsymbol{\nu}}\operatorname{sgn}(\boldsymbol{\mu})\operatorname{sgn}(\boldsymbol{\nu})K(\boldsymbol{\mu},\boldsymbol{\nu})\right)\left(\frac{\mathsf{snr}}{n_\mathsf{T}}\right)^{\sum_{t=1}^k \mu_t+\nu_t} \quad (79)$$

where $\sum_{t=1}^k \mu_t+\nu_t = 2\sum_{t=1}^k t = k(k+1)$ and the constant $K(\cdot)$ is given by

$$K(\boldsymbol{\mu},\boldsymbol{\nu}) = \prod_{t=1}^k \Gamma(\mu_t+\nu_t). \quad (80)$$

Hence, $\sum_{\boldsymbol{\mu},\boldsymbol{\nu}}\operatorname{sgn}(\boldsymbol{\mu})\operatorname{sgn}(\boldsymbol{\nu})K(\boldsymbol{\mu},\boldsymbol{\nu})$ has the form of a determinant [43, eq. (38)]

$$\sum_{\boldsymbol{\mu},\boldsymbol{\nu}}\operatorname{sgn}(\boldsymbol{\mu})\operatorname{sgn}(\boldsymbol{\nu})K(\boldsymbol{\mu},\boldsymbol{\nu}) = k!\sum_{\boldsymbol{\mu}}\operatorname{sgn}(\boldsymbol{\mu})\prod_{t=1}^k\Gamma(\mu_t+t) = k!\prod_{t=1}^k(t-1)!t! \quad (81)$$

where last equality comes from [44, eq. (4.5)]. Finally, the asymptotic behavior of $\Lambda_{(k)}(\mathsf{r},\mathsf{snr})$ is

$$\Lambda_{(k)}(\mathsf{r},\mathsf{snr}) \sim |\mathbf{A}(k)|\left(\prod_{t=1}^k(t-1)!t!\right)\left(\frac{\mathsf{snr}^{\mathsf{r}\zeta(k+1)}}{\zeta(k+1)}\right)\left(\frac{\mathsf{snr}}{n_\mathsf{T}}\right)^{k(k+1)}. \quad (82)$$

A.ii Proof of Theorem 1.ii (Semicorrelated Rayleigh):

Under semicorrelated Rayleigh fading, either $\mathbf{H}\mathbf{H}^\dagger$ or $\mathbf{H}^\dagger\mathbf{H}$ is correlated central Wishart distributed [25, Sec II]. The joint pdf of the nonzero ordered eigenvalues $\boldsymbol{\lambda} = (\lambda_1,\ldots,\lambda_n)$ is then given by [43, eq. (17)]

$$f_{\boldsymbol{\lambda}}(\boldsymbol{\lambda}) = K_{m,n}(\boldsymbol{\sigma})|\mathbf{E}(\boldsymbol{\lambda},\boldsymbol{\sigma})||\mathbf{V}(\boldsymbol{\lambda})|\prod_{t=1}^n \lambda_t^{m-n} \quad (83)$$

where the normalization constant $K_{m,n}(\boldsymbol{\sigma})$ is

$$K_{m,n}(\boldsymbol{\sigma}) = \prod_{t=1}^n \frac{1}{\sigma_t^m(m-t)!}\prod_{i<j}^n\left(\frac{\sigma_i\sigma_j}{\sigma_j-\sigma_i}\right) \quad (84)$$

and matrix $\mathbf{E}(\boldsymbol{\lambda},\boldsymbol{\sigma})$ $(n\times n)$ is defined as

$$[\mathbf{E}(\boldsymbol{\lambda},\boldsymbol{\sigma})]_{u,v} = e^{-\lambda_v/\sigma_u}. \quad (85)$$

Let us now introduce the ordered variables $\alpha_1 \geq \cdots \geq \alpha_n \geq 1$ defined in (45) so that the outage probability in (16) can be rewritten in terms of $\boldsymbol{\alpha}$ as in (46). However, the joint pdf of $\boldsymbol{\alpha}$ is in this case given by

$$f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) = K_{m,n}(\boldsymbol{\sigma})\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)^{mn-n(n-1)/2} e^{\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\sum_{t=1}^n\left(\frac{1}{\sigma_t}\right)} |\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})|\prod_{i<j}^n(\alpha_j-\alpha_i)\prod_{t=1}^n(\alpha_t-1)^{m-n} \quad (86)$$

where matrix $\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})$ $(n\times n)$ is defined as

$$[\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})]_{u,v} = e^{-\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\left(\frac{\alpha_u}{\sigma_v}\right)}. \quad (87)$$

As we state in the proof of Theorem 1.i, not all $\boldsymbol{\alpha}\in\mathcal{A}(\mathsf{r},\mathsf{snr})$ contribute to the high-SNR behavior of $\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr})$, but only those $\boldsymbol{\alpha}\in\mathcal{A}^\star(\mathsf{r},\mathsf{snr})$ where (49) is satisfied and the following asymptotic equivalences hold:

$$\prod_{i=1}^k\prod_{j=k+1}^n(\alpha_j-\alpha_i) \sim (-1)^{k(n-k)}\prod_{t=1}^k\alpha_t^{n-k} \quad (88)$$

$$\prod_{t=1}^k(\alpha_t-1)^{m-n} \sim \prod_{t=1}^k\alpha_t^{m-n} \quad (89)$$

$$e^{\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\sum_{t=1}^n\left(\frac{1}{\sigma_t}\right)} \sim 1 \quad (90)$$

since $\{\alpha_t\}_{t=1}^k$ are $O(\mathsf{snr})$, whereas $\{\alpha_t\}_{t=k+1}^n$ are $o(\mathsf{snr})$. Furthermore in Appendix A.ii.1, we show that

$$|\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})| \sim \left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)^{(n-k)(n-k-1)/2}|\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma})|\prod_{k<i<j}^n(\alpha_j-\alpha_i) \quad (91)$$

where $\boldsymbol{\alpha}_k = (\alpha_1,\ldots,\alpha_k)$ and matrix $\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma})$ is defined in (108). Then, expanding $\prod_{i<j}^n(\alpha_j-\alpha_i)$ as in (53) and using the previous asymptotic equivalences, the high-SNR behavior of the outage probability is

$$\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr}) \sim K_{m,n}(\boldsymbol{\sigma})(-1)^{k(n-k)}\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)^{mn-k(2n-k-1)/2} \int_{\mathcal{A}^\star(\mathsf{r},\mathsf{snr})}|\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma})|\prod_{i<j}^k(\alpha_j-\alpha_i)\prod_{t=1}^k\alpha_t^{m-k} \prod_{k<i<j}^n(\alpha_j-\alpha_i)^2\prod_{t=k+1}^n(\alpha_t-1)^{m-n}d\boldsymbol{\alpha}. \quad (92)$$

Similarly to the proof of Theorem 1.i, let us introduce

$$\Lambda_{(k)}(\mathsf{r},\mathsf{snr}) = \int_{\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr})}|\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma})|\Lambda^{(k)}(\boldsymbol{\alpha}_k;\mathsf{r},\mathsf{snr}) \prod_{i<j}^k(\alpha_j-\alpha_i)\prod_{t=1}^k\alpha_t^{m-k}d\boldsymbol{\alpha}_k \quad (93)$$

where $\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr})$ is defined in (57) and $\Lambda^{(k)}(\boldsymbol{\alpha}_k;\mathsf{r},\mathsf{snr})$ and its asymptotic behavior are given in (56) and (60), respectively. Now, we can rewrite (92) as

$$\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr}) \sim K_{m,n}(\boldsymbol{\sigma})(-1)^{k(n-k)} \left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{mn-k(2n-k-1)/2} \Lambda_{(k)}(\mathsf{r},\mathsf{snr}) \quad (94)$$

where the asymptotic behavior of $\Lambda_{(k)}(\mathsf{r},\mathsf{snr})$, when substituting (60) back in (93), is shown in Appendix A.ii.2 to be given by

$$\Lambda_{(k)}(\mathsf{r},\mathsf{snr}) \sim |\mathbf{A}(k)||\mathbf{A}_2(k,\boldsymbol{\sigma})| \left(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\right)\left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{(n-k-1)k-k(k+1)/2} \quad (95)$$

where $\zeta(t) = m+n-t$ and matrices $\mathbf{A}(k)$ and $\mathbf{A}_2(k,\boldsymbol{\sigma})$ are defined in (74) and (25), respectively. Finally, combining (95) with (94), the array gain $\mathsf{a}_2(\mathsf{r})$ provided in Theorem 1 for the semicorrelated Rayleigh channel follows. ☐

A.ii.1) Asymptotic Expansion of $|\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})|$*:* The objective of this appendix is to obtain an asymptotic expansion of $|\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})|$ defined in (87) as $\mathsf{snr}\to\infty$ and when $\boldsymbol{\alpha}$ satisfies (49), i.e., when $\{\alpha_t\}_{t=k+1}^{n} = o(\mathsf{snr})$.

First, in order to separate the columns with functions of $\alpha_1,\ldots,\alpha_k$ from the ones including functions of $\alpha_{k+1},\ldots,\alpha_n$, we use the Laplace expansion of the determinant [34, Sec 0.3.1]:

$$|\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})| = \sum_{\boldsymbol{\iota}\in\mathcal{I}}(-1)^{\sum_{t=1}^{k}(\iota_t+t)} |\mathbf{F}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma};\mathsf{snr})||\mathbf{G}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\mathsf{snr})| \quad (96)$$

where $\boldsymbol{\alpha}^k = (\alpha_{k+1},\ldots,\alpha_n)$ and the summation over $\boldsymbol{\iota} = (\iota_1,\ldots,\iota_n)$ is for all permutations of integers $(1,\ldots,n)$ such that $(\iota_1 < \cdots < \iota_k)$ and $(\iota_{k+1} < \cdots < \iota_n)$, and $\mathbf{F}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma};\mathsf{snr})$ and $\mathbf{G}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\mathsf{snr})$ are defined as

$$[\mathbf{F}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma};\mathsf{snr})]_{u,v} = e^{-\left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)\left(\frac{\alpha_u}{\sigma_{\iota_v}}\right)} \quad (97)$$

for $u,v = 1,\ldots,k$, and

$$[\mathbf{G}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\mathsf{snr})]_{u,v} = e^{-\left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)\left(\frac{\alpha_{k+u}}{\sigma_{\iota_{k+v}}}\right)} \quad (98)$$

for $u,v = 1,\ldots,n-k$. Then, the asymptotic expansion of $|\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})|$ follows from obtaining the first-order Taylor expansion of $|\mathbf{G}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\mathsf{snr})|$ around $\eta=0$ with $\eta = 1/\mathsf{snr}$:

$$|\mathbf{G}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\eta=0)| \sim \left(\frac{d^{r^\star}}{d\eta^{r^\star}}|\mathbf{G}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\eta=0)|\right)\frac{\eta^{r^\star}}{r^\star!} \quad (99)$$

where $r^\star$ denotes the minimum r such that the rth derivative of $|\mathbf{G}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\mathsf{snr})|$ evaluated at $\eta = 0$ does not equal 0. Using [45, eq. (10)], it follows that

$$\frac{d^r}{d\eta^r}|\mathbf{G}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\eta=0)| = \sum_{\boldsymbol{r}}\frac{r!}{r_1!\cdots r_{n-k}!}|\mathbf{G}^{(\boldsymbol{\iota},\boldsymbol{r})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\eta=0)| \quad (100)$$

where the summation over $\boldsymbol{r} = (r_1,\ldots,r_{n-k})$ is for all $\boldsymbol{r}$ such that $r_v \in \mathbb{N}\cup\{0\}$ and $\sum_{v=1}^{n-k} r_v = r$, and matrix $\mathbf{G}^{(\boldsymbol{\iota},\boldsymbol{r})}(\boldsymbol{\alpha}^k;\eta)$ $\big((n-k)\times(n-k)\big)$ is defined as

$$\begin{aligned}[\mathbf{G}^{(\boldsymbol{\iota},\boldsymbol{r})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\eta=0)]_{u,v} &= \frac{d^{r_v}}{d\eta^{r_v}}\left(e^{-n_{\mathsf{T}}\eta\left(\frac{\alpha_{k+u}}{\sigma_{\iota_{k+v}}}\right)}\right)\Big|_{\eta=0} \\ &= n_{\mathsf{T}}^{r_v}\left(\frac{-\alpha_{k+u}}{\sigma_{\iota_{k+v}}}\right)^{r_v}. \end{aligned} \quad (101)$$

This shows that all r_v in the set $\{r_v\}_{u=1}^{n-k}$ have to be different to avoid having linearly dependent columns. Thus, the minimum $r = \sum_{v=1}^{n-k} r_v$ that leads to a nonzero determinant has

$$r_v = \nu_u - 1 \quad (102)$$

where $\boldsymbol{\nu} = (\nu_1,\ldots,\nu_{n-k})$ is a permutation of integers $(1,\ldots,n-k)$, and $r^\star$ is equal to

$$r^\star = (n-k)(n-k-1)/2. \quad (103)$$

The asymptotic equivalence in (99) results then in

$$|\mathbf{G}^{(\boldsymbol{\iota})}(\boldsymbol{\alpha}^k,\boldsymbol{\sigma};\mathsf{snr})| \sim \left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{(n-k)(n-k-1)/2} \sum_{\boldsymbol{\mu},\boldsymbol{\nu}}\mathrm{sgn}(\boldsymbol{\mu})\prod_{u=1}^{n-k}\frac{1}{\Gamma(u)}\left(\frac{-\alpha_{k+\mu_u}}{\sigma_{\iota_{k+u}}}\right)^{\nu_u-1} \quad (104)$$

where the determinant can be simplified as [35, Proposition 1]

$$\begin{aligned}&\sum_{\boldsymbol{\mu},\boldsymbol{\nu}}\mathrm{sgn}(\boldsymbol{\mu})\prod_{u=1}^{n-k}\frac{1}{\Gamma(u)}\left(\frac{-\alpha_{k+\mu_u}}{\sigma_{\iota_{k+u}}}\right)^{\nu_u-1} \\ &= \sum_{\boldsymbol{\mu},\boldsymbol{\nu}}\mathrm{sgn}(\boldsymbol{\mu})\mathrm{sgn}(\boldsymbol{\nu})\prod_{u=1}^{n-k}\frac{1}{\Gamma(u)}\left(\frac{-\alpha_{k+\mu_u}}{\sigma_{\iota_{k+\nu_u}}}\right)^{u-1} \quad (105) \\ &= \prod_{k<i<j}^{n}(\alpha_j-\alpha_i)\sum_{\boldsymbol{\nu}}\mathrm{sgn}(\boldsymbol{\nu})\prod_{u=1}^{n-k}\frac{1}{\Gamma(u)}\left(\frac{-1}{\sigma_{\iota_{k+\nu_u}}}\right)^{u-1}. \quad (106)\end{aligned}$$

Finally, undoing the Laplace expansion, it follows that

$$|\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})| \sim \left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{(n-k)(n-k-1)/2}|\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma})|\prod_{k<i<j}^{n}(\alpha_j-\alpha_i) \quad (107)$$

where matrix $\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma})$ $(n\times n)$ is defined as

$$[\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma})]_{u,v}=\begin{cases} e^{-\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\left(\frac{\alpha_u}{\sigma_v}\right)} & 1\le u\le k\\ \frac{1}{\Gamma(u-k)}\left(\frac{-1}{\sigma_v}\right)^{u-k-1} & k<u\le n.\end{cases} \quad (108)$$

1) Integral $\Lambda_{(k)}(\mathsf{r},\mathsf{snr})$*:* The objective of this appendix is to derive the asymptotic behavior of

$$\Lambda_{(k)}(\mathsf{r},\mathsf{snr})\sim|\mathbf{A}(k)|\Big(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\Big)\int_{\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr})}|\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma})| \prod_{i=1}^{k}\alpha_i^{-(n-k-1)}\prod_{i<j}^{k}(\alpha_j-\alpha_i)d\boldsymbol{\alpha}_k \quad (109)$$

where $\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr})$ is defined in (58), $\zeta(t)=m+n-t$, and matrix $\mathbf{A}(k)$ is given in (74).

First, we expand the determinants

$$\prod_{i<j}^{k}(\alpha_j-\alpha_i)=\sum_{\boldsymbol{\nu}}\mathrm{sgn}(\boldsymbol{\nu})\prod_{t=1}^{k}\alpha_t^{\nu_t-1} \quad (110)$$

$$|\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\sigma})|=\sum_{\boldsymbol{\mu}}\mathrm{sgn}(\boldsymbol{\mu})\prod_{t=1}^{k}e^{-\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\left(\frac{\alpha_t}{\sigma_{\mu_t}}\right)} \prod_{t=k+1}^{n}\frac{1}{\Gamma(t-k)}\Big(\frac{-1}{\sigma_{\mu_t}}\Big)^{t-k-1} \quad (111)$$

where the summation over $\boldsymbol{\nu}=(\nu_1,\ldots,\nu_k)$ is for all permutations of integers $(1,\ldots,k)$ and the summation over $\boldsymbol{\mu}=(\mu_1,\ldots,\mu_n)$ is for all permutations of integers $(1,\ldots,n)$. Substituting back in (109) and exploiting the symmetry of the integrand, we have that

$$\Lambda_{(k)}(\mathsf{r},\mathsf{snr})\sim\frac{|\mathbf{A}(k)|}{k!}\Big(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\Big)\int_1^{\mathsf{snr}^\mathsf{r}}\cdots\int_1^{\left(\frac{\mathsf{snr}^\mathsf{r}}{\prod_{u=1}^{k-1}\alpha_u}\right)} \sum_{\boldsymbol{\mu},\boldsymbol{\nu}}\mathrm{sgn}(\boldsymbol{\mu})\mathrm{sgn}(\boldsymbol{\nu})\prod_{t=1}^{k}e^{-\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\left(\frac{\alpha_t}{\sigma_{\mu_t}}\right)}\alpha_t^{\nu_t-(n-k)} \prod_{t=k+1}^{n}\frac{1}{\Gamma(t-k)}\Big(\frac{-1}{\sigma_{\mu_t}}\Big)^{t-k-1}d\boldsymbol{\alpha}_k \quad (112)$$

$$=|\mathbf{A}(k)|\Big(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\Big)\int_1^{\mathsf{snr}^\mathsf{r}}\cdots\int_1^{\left(\frac{\mathsf{snr}^\mathsf{r}}{\prod_{u=1}^{k-1}\alpha_u}\right)} \sum_{\boldsymbol{\mu}}\mathrm{sgn}(\boldsymbol{\mu})\prod_{t=1}^{k}e^{-\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\left(\frac{\alpha_t}{\sigma_{\mu_t}}\right)}\alpha_t^{t-(n-k)} \prod_{t=k+1}^{n}\frac{1}{\Gamma(t-k)}\Big(\frac{-1}{\sigma_{\mu_t}}\Big)^{t-k-1}d\boldsymbol{\alpha}_k. \quad (113)$$

Now, observing that

$$\int_1^{\left(\frac{\mathsf{snr}^\mathsf{r}}{\prod_{u=1}^{t-1}\alpha_u}\right)} e^{-\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\left(\frac{\alpha_t}{\sigma_{\mu_t}}\right)}\alpha_t^{t-(n-k)}d\alpha_t\sim \int_1^{\infty} e^{-\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\left(\frac{\alpha_t}{\sigma_{\mu_t}}\right)}\alpha_t^{t-(n-k)}d\alpha_t \quad (114)$$

it follows that

$$\Lambda_{(k)}(\mathsf{r},\mathsf{snr})\sim|\mathbf{A}(k)||\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})|\Big(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\Big)$$

where matrix $|\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})|$ $(n\times n)$ is defined as

$$[\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})]_{u,v}=\begin{cases}\int_1^\infty e^{-\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)\left(\frac{\alpha}{\sigma_u}\right)}\alpha^{v-(n-k)}d\alpha & 1\le v\le n\\ \frac{1}{\Gamma(v-k)}\left(\frac{-1}{\sigma_u}\right)^{v-k-1} & k<v\le n\end{cases} \quad (115)$$

and can be rewritten as shown in (116) at the bottom of the page, where $\mathrm{E}.(\cdot)$ denotes the generalized exponential integral [42, eq. (5.1.4)] and $\Gamma(\cdot,\cdot)$ denotes the upper incomplete gamma function [42, eq. (6.5.3)].

Now, the original problem reduces to finding an asymptotic expansion for the determinant $|\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})|$. Let us concentrate on the $\min(n-k-1,k)$ first columns, which satisfy [42, (5.1.12)]

$$[\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})]_{u,v}=\frac{1}{\Gamma(n-k-v)}\Big(\frac{-n_\mathsf{T}}{\mathsf{snr}\sigma_u}\Big)^{n-k-v-1}\Big(\psi(n-k-v)-\ln\Big(\frac{n_\mathsf{T}}{\mathsf{snr}\sigma_u}\Big)\Big) -\sum_{t=0,t\neq n-k-v-1}^{\infty}\Big(\frac{1}{(t-(n-k)+v)t!}\Big)\Big(\frac{-n_\mathsf{T}}{\mathsf{snr}\sigma_u}\Big)^{t} \quad (117)$$

where $\psi(\cdot)$ denotes the digamma function [42, eq. (6.3.2)]. Now, expanding $|\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})|$ over the vth $(1\le v\le\min(n-k-1,k))$ column using the multilinear property of determinants [34, Sec. 0.3.6], it follows that

$$|\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})|=\sum_{t=0}^{\infty}|\mathbf{E}_{v,t}(\boldsymbol{\sigma},\mathsf{snr})| \quad (118)$$

where $\mathbf{E}_{v,t}(\boldsymbol{\sigma},\mathsf{snr})$ equals $\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})$ except for the vth column which is given by (119) at the bottom of the next page. Let us now remove from the infinite summation in (118) the terms that lead to a zero determinant

$$|\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})|=|\mathbf{E}_{v,n-k-v-1}(\boldsymbol{\sigma},\mathsf{snr})|+\sum_{t=n-k}^{\infty}|\mathbf{E}_{v,t}(\boldsymbol{\sigma},\mathsf{snr})| \quad (120)$$

$$[\mathbf{E}(\boldsymbol{\sigma},\mathsf{snr})]_{u,v}=\begin{cases}\mathrm{E}_{(n-k)-v}\left(\frac{n_\mathsf{T}}{\mathsf{snr}\sigma_u}\right) & 1\le v\le\min(n-k-1,k)\\ \Gamma\left(v-(n-k)+1,\left(\frac{n_\mathsf{T}}{\mathsf{snr}\sigma_u}\right)\right)\left(\frac{n_\mathsf{T}}{\mathsf{snr}\sigma_u}\right)^{(n-k)-v-1} & n-k\le v\le k\\ \frac{1}{\Gamma(v-k)}\left(\frac{-1}{\sigma_u}\right)^{v-k-1} & k<v\le n\end{cases} \quad (116)$$

since the $(t+1)$th and $(k+t)$th columns of $\mathbf{E}_{v,t}(\boldsymbol{\sigma}, \mathsf{snr})$ are linearly dependent for $t = 0, \ldots, n-k-1$. In addition, the term with highest SNR exponent, $|\mathbf{E}_{v,n-k-v-1}(\boldsymbol{\sigma}, \mathsf{snr})|$, can be further simplified using again the multilinear property to remove the summands that are linearly dependent to the $(n-k-v)$th column:

$$|\mathbf{E}(\boldsymbol{\sigma}, \mathsf{snr})| \sim |\mathbf{E}_{v,n-k-v-1}(\boldsymbol{\sigma}, \mathsf{snr})| = |\mathbf{E}_v(\boldsymbol{\sigma}, \mathsf{snr})| \quad (121)$$

where $\mathbf{E}_v(\boldsymbol{\sigma}, \mathsf{snr})$ equals $\mathbf{E}(\boldsymbol{\sigma}, \mathsf{snr})$ except for the vth column which is given by

$$[\mathbf{E}_v(\boldsymbol{\sigma}, \mathsf{snr})]_{u,v} = \frac{1}{\Gamma(n-k-v)}\left(\frac{-n_\mathsf{T}}{\mathsf{snr}\sigma_u}\right)^{n-k-v-1} \ln(\sigma_u). \quad (122)$$

Performing this procedure recursively for $v = 1, \ldots, \min(n-k-1, k)$, and substituting the incomplete gamma functions by their asymptotic equivalent, $\Gamma(a,x) \sim \Gamma(x)$ as $x \to 0$ [42, Sec. 6.5], we have that

$$|\mathbf{E}(\boldsymbol{\sigma}, \mathsf{snr})| \sim \sum_{\boldsymbol{\mu}} \mathrm{sgn}(\boldsymbol{\mu}) \prod_{t=1}^{\min(n-k-1,k)} \frac{1}{\Gamma(n-k-t)}\left(\frac{-n_\mathsf{T}}{\mathsf{snr}\sigma_{\mu_t}}\right)^{(n-k)-t-1} \ln(\sigma_{\mu_t}) \prod_{t=n-k}^{k} \Gamma(t-(n-k)+1)\left(\frac{n_\mathsf{T}}{\mathsf{snr}\sigma_{\mu_t}}\right)^{(n-k)-t-1} \prod_{t=k+1}^{n} \frac{1}{\Gamma(t-k)}\left(\frac{-1}{\sigma_{\mu_t}}\right)^{t-k-1}. \quad (123)$$

Hence, it finally results that

$$|\mathbf{E}(\boldsymbol{\sigma}, \mathsf{snr})| \sim |\mathbf{A}_2(k, \boldsymbol{\sigma})| \left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)^{(n-k-1)k-k(k+1)/2} \quad (124)$$

$$\Lambda_{(k)}(\mathsf{r}, \mathsf{snr}) \sim |\mathbf{A}(k)||\mathbf{A}_2(k, \boldsymbol{\sigma})| \left(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\right)\left(\frac{n_\mathsf{T}}{\mathsf{snr}}\right)^{(n-k-1)k-k(k+1)/2} \quad (125)$$

and from (123), we know that matrix $\mathbf{A}_2(k, \boldsymbol{\sigma})$ $(n \times n)$ is defined as in (25).

A.iii. Proof of Theorem 1.iii (Uncorrelated Rician):

Under uncorrelated Rician fading, either $(\mathsf{K}+1)\mathbf{H}\mathbf{H}^\dagger$ or $(\mathsf{K}+1)\mathbf{H}^\dagger\mathbf{H}$ is uncorrelated noncentral Wishart distributed [25, Sec. II]. The joint pdf of the nonzero ordered eigenvalues $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)$ of $\mathbf{H}^\dagger\mathbf{H}$ is given by [46, eq. (45)]

$$f_{\boldsymbol{\lambda}}(\boldsymbol{\lambda}) = K_{m,n}(\boldsymbol{\omega})(\mathsf{K}+1)^{(2m-n+1)n/2} |\mathbf{E}(\boldsymbol{\lambda}, \boldsymbol{\omega})||\mathbf{V}(\boldsymbol{\lambda})| \prod_{t=1}^{n} e^{-(\mathsf{K}+1)\lambda_t}\lambda_t^{m-n} \quad (126)$$

where the normalization constant $K_{m,n}(\boldsymbol{\omega})$ is

$$K_{m,n}(\boldsymbol{\omega}) = \frac{e^{-\sum_{t=1}^{n}\omega_t}}{\Gamma(m-n+1)^n} \prod_{i<j}^{n} \frac{1}{(\omega_j - \omega_i)} = \frac{e^{-mn\mathsf{K}}}{\Gamma(m-n+1)^n} \prod_{i<j}^{n} \frac{1}{(\omega_j - \omega_i)} \quad (127)$$

and matrix $\mathbf{E}(\boldsymbol{\lambda}, \boldsymbol{\omega})$ $(n \times n)$ is defined as

$$[\mathbf{E}(\boldsymbol{\lambda}, \boldsymbol{\omega})]_{u,v} = {}_0F_1\big(m-n+1; (\mathsf{K}+1)\omega_v\lambda_u\big) \quad (128)$$

where ${}_0F_1(\cdot;\cdot)$ denotes the Bessel-type hypergeometric function [42, eq. (9.6.47)]. Let us now introduce the ordered variables $\alpha_1 \geq \cdots \geq \alpha_n > 1$ defined in (45) so that the outage probability in (16) can be rewritten in terms of $\boldsymbol{\alpha}$ as in (46). The joint pdf of $\boldsymbol{\alpha}$ is now given by

$$f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) = K_{m,n}(\boldsymbol{\omega})\left(\frac{n_\mathsf{K}}{\mathsf{snr}}\right)^{mn-(n-1)n/2} e^{n(\frac{n_\mathsf{K}}{\mathsf{snr}})} |\mathbf{E}(\boldsymbol{\alpha}, \boldsymbol{\omega})| \prod_{i<j}^{n}(\alpha_j - \alpha_i) \prod_{t=1}^{n} e^{-(\frac{n_\mathsf{K}}{\mathsf{snr}})\alpha_t}(\alpha_t - 1)^{m-n} \quad (129)$$

with $n_\mathsf{K} = (\mathsf{K}+1)n_\mathsf{T}$ and matrix $\mathbf{E}(\boldsymbol{\alpha}, \boldsymbol{\omega})$ $(n \times n)$ defined as

$$[\mathbf{E}(\boldsymbol{\alpha}, \boldsymbol{\omega})]_{u,v} = {}_0F_1\left(m-n+1; \omega_v\left(\frac{n_\mathsf{K}}{\mathsf{snr}}\right)(\alpha_u - 1)\right). \quad (130)$$

Now, using the same techniques as in the proofs of Theorem 1.ii and expanding $|\mathbf{E}(\boldsymbol{\alpha}, \boldsymbol{\omega})|$ as shown in Appendix A.iii.1, the high-SNR behavior of outage probability in (46) satisfies

$$\mathsf{P}_{\mathsf{out}}(\mathsf{r}, \mathsf{snr}) \sim K_{m,n}(\boldsymbol{\omega})(-1)^{k(n-k)} \left(\frac{n_\mathsf{K}}{\mathsf{snr}}\right)^{mn-k(2n-k-1)/2} \Lambda_{(k)}(\mathsf{r}, \mathsf{snr}) \quad (131)$$

with

$$\Lambda_{(k)}(\mathsf{r}, \mathsf{snr}) = \int_{\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr})} |\mathbf{E}(\boldsymbol{\alpha}_k, \boldsymbol{\omega})|\Lambda^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr}) \prod_{i<j}^{k}(\alpha_j - \alpha_i) \prod_{t=1}^{k} e^{-(\frac{n_\mathsf{K}}{\mathsf{snr}})\alpha_t}\alpha_t^{m-k} d\boldsymbol{\alpha}_k \quad (132)$$

where $\mathbf{E}(\boldsymbol{\alpha}_k, \boldsymbol{\omega})$ is given in (137) and $\Lambda^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr})$ and $\mathcal{A}_{(k)}(\mathsf{r}, \mathsf{snr})$ are defined in (56) and (57), respectively. Then, the asymptotic behavior of $\Lambda_{(k)}(\mathsf{r}, \mathsf{snr})$, when substituting the asymptotic equivalence for $\Lambda^{(k)}(\boldsymbol{\alpha}_k; \mathsf{r}, \mathsf{snr})$ obtained in Appendix A.i.1, is shown in Appendix A.iii.2 to be given by

$$\Lambda_{(k)}(\mathsf{r}, \mathsf{snr}) \sim \Gamma(m-n+1)^n |\mathbf{A}(k)||\mathbf{A}_3(k, \boldsymbol{\omega})| \left(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\right)\left(\frac{n_\mathsf{K}}{\mathsf{snr}}\right)^{(n-k-1)k-k(k+1)/2} \quad (133)$$

$$[\mathbf{E}_{v,t}(\boldsymbol{\sigma}, \mathsf{snr})]_{u,v} = \begin{cases} \frac{1}{\Gamma(n-k-v)}\left(\frac{-n_\mathsf{T}}{\mathsf{snr}\sigma_u}\right)^{n-k-v-1}\left(\psi(n-k-v) - \ln\left(\frac{n_\mathsf{T}}{\mathsf{snr}\sigma_u}\right)\right) & t = n-k-v-1 \\ \left(\frac{-1}{(t-(n-k)+v)t!}\right)\left(\frac{-n_\mathsf{T}}{\mathsf{snr}\sigma_u}\right)^{t} & t \neq n-k-v-1. \end{cases} \quad (119)$$

where $\zeta(t) = m + n - t$ and matrices $\mathbf{A}(k)$ and $\mathbf{A}_3(k,\boldsymbol{\omega})$ are defined in (74) and (144), respectively. Finally, combining (133) with (131), the array gain $\mathsf{a}_3(\mathsf{r})$ provided Theorem 1 for the uncorrelated Rician channel follows. □

A.iii.1) Asymptotic Expansion of $|\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\omega})|$*:* The objective of this appendix is to obtain an asymptotic expansion of $|\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\omega})|$ defined in (130) when $\boldsymbol{\alpha}$ satisfies (49) and $\mathsf{snr} \to \infty$. We can use exactly the same procedure as in the asymptotic expansion of $|\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\sigma})|$ in Appendix A.ii.1.

Observing that [42, eq. (9.6.47) and eq. (9.6.10)]

$$ {}_0F_1(a+1;b\eta) = \Gamma(a+1)\sum_{t=0}^{\infty}\frac{b^t}{\Gamma(a+t+1)}\left(\frac{\eta^t}{t!}\right) \quad (134) $$

it follows that

$$ \frac{d^{r_v}}{d\eta^{r_v}}\left[{}_0F_1\Big(m-n+1;(n_{\mathsf{K}}\eta\omega_{\iota_{k+v}})(\alpha_{k+u}-1)\Big)\right]\Big|_{\eta=0} = \frac{(n_{\mathsf{K}}\omega_{\iota_{k+v}})^{r_v}}{\Gamma(m-n+r_v+1)}(\alpha_{k+u}-1)^{r_v}. \quad (135) $$

Thus, we finally have that

$$ |\mathbf{E}(\boldsymbol{\alpha},\boldsymbol{\omega})| \sim \left(\frac{n_{\mathsf{K}}}{\mathsf{snr}}\right)^{(n-k)(n-k-1)/2} \prod_{k<i<j}^{n}(\alpha_j-\alpha_i)|\mathbf{E}(\alpha_1,\ldots,\alpha_k,\boldsymbol{\omega})| \quad (136) $$

where matrix $\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\omega})$ $(n\times n)$ is defined as

$$ [\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\omega})]_{u,v} = \begin{cases} {}_0F_1\big(m-n+1;\omega_v\big(\frac{n_{\mathsf{K}}}{\mathsf{snr}}\big)(\alpha_u-1)\big) & 1\le u\le k \\ \frac{\Gamma(m-n+1)}{\Gamma(u-k)\Gamma(m-n+u-k)}\omega_v^{u-k-1} & k<u\le n. \end{cases} \quad (137) $$

A.iii.2) Integral $\Lambda_{(k)}(\mathsf{r},\mathsf{snr})$*:* The objective of this appendix is to derive the asymptotic behavior of

$$ \Lambda_{(k)}(\mathsf{r},\mathsf{snr}) \sim |\mathbf{A}(k)|\left(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\right)\int_{\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr})}|\mathbf{E}(\boldsymbol{\alpha}_k,\boldsymbol{\omega})| \prod_{i=1}^{k}\alpha_i^{-(n-k-1)}\prod_{i<j}^{k}(\alpha_j-\alpha_i)d\boldsymbol{\alpha}_k \quad (138) $$

where $\mathcal{A}_{(k)}(\mathsf{r},\mathsf{snr})$ is defined in (58), $\zeta(t) = m+n-t$, and matrix $\mathbf{A}(k)$ is given in (74). Following the same procedure as in Appendix A.ii.2, we have that

$$ \Lambda_{(k)}(\mathsf{r},\mathsf{snr}) \sim |\mathbf{A}(k)||\mathbf{E}(\boldsymbol{\omega},\mathsf{snr})|\left(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\right) \quad (139) $$

where matrix $\mathbf{E}(\boldsymbol{\omega},\mathsf{snr})$ $(n\times n)$ is defined in (140) at the bottom of the page. Note that for $v=1,\ldots,k$, it holds

$$ [\mathbf{E}(\boldsymbol{\omega},\mathsf{snr})]_{u,v} \sim \sum_{i=0}^{\infty}\frac{\Gamma(m-n+1)}{\Gamma(m-n+i+1)}\left(\frac{n_{\mathsf{K}}}{\mathsf{snr}}\right)^i\left(\frac{\omega_u^i}{i!}\right) \int_1^{\infty}e^{-(\frac{n_{\mathsf{K}}}{\mathsf{snr}})\alpha}\alpha^{v+i-(n-k)}d\alpha \quad (141) $$

where we have used the series expansion in (134). Now, removing the linearly dependent terms as in Appendix A.ii.2, we have that

$$ |\mathbf{E}(\boldsymbol{\omega},\mathsf{snr})| \sim \Gamma(m-n+1)^n|\mathbf{A}_3(k,\boldsymbol{\omega})| \left(\frac{n_{\mathsf{K}}}{\mathsf{snr}}\right)^{(n-k-1)k-k(k+1)/2} \quad (142) $$

where matrix $\mathbf{A}_3(k,\boldsymbol{\omega})$ is defined as

$$ [\mathbf{A}_3(k,\boldsymbol{\omega})]_{u,v} = \begin{cases} \sum_{i=n-k}^{\infty}\frac{\Gamma(v+i-(n-k)+1)}{\Gamma(m-n+i+1)}\frac{\omega_u^i}{i!} & 1\le v\le k \\ \frac{1}{\Gamma(v-k)\Gamma(m-n+v-k)}\omega_u^{v-k-1} & k<v\le n \end{cases} \quad (143) $$

or, equivalently, as

$$ [\mathbf{A}_3(k,\boldsymbol{\omega})]_{u,v} = \begin{cases} \frac{\Gamma(v+1)}{\Gamma(m-k+1)\Gamma(n-k+1)}\mathcal{F}_v(k,\omega_u) & 1\le v\le k \\ \frac{1}{\Gamma(v-k)\Gamma(m-n+v-k)}\omega_u^{v-k-1} & k<v\le n \end{cases} \quad (144) $$

where we have introduced

$$ \mathcal{F}_v(k,\omega_u) = \frac{\Gamma(m-k+1)\Gamma(n-k+1)}{\Gamma(v+1)} \sum_{i=(n-k)}^{\infty}\frac{\Gamma(v+i-(n-k)+1)}{\Gamma(m-n+i+1)}\left(\frac{\omega_u^i}{i!}\right) \quad (145) $$

$$ = \omega_u^{n-k}{}_2F_2(v+1,1;m-k+1,n-k+1;\omega_u) \quad (146) $$

with ${}_2F_2(\cdot;\cdot)$ denoting the generalized hypergeometric function [33, eq. (9.14.1]. Hence, it finally follows that

$$ \Lambda_{(k)}(\mathsf{r},\mathsf{snr}) \sim \Gamma(m-n+1)^n|\mathbf{A}(k)||\mathbf{A}_3(k,\boldsymbol{\omega})| \left(\frac{\mathsf{snr}^{\mathsf{r}\zeta(2k+1)}}{\zeta(2k+1)}\right)\left(\frac{n_{\mathsf{K}}}{\mathsf{snr}}\right)^{(n-k-1)k-k(k+1)/2}. \quad (147) $$

Appendix B
Proof of Theorem 2

The outage probability $\mathsf{P}_{\mathsf{out}}(\mathsf{R},\mathsf{snr})$ in (16) when R satisfies (34) can be expressed in terms of the ordered variables $\boldsymbol{\alpha} = (\alpha_1,\ldots,\alpha_n)$ introduced in (45) as

$$ \mathsf{P}_{\mathsf{out}}(\mathsf{R},\mathsf{snr}) \sim \mathsf{Pr}\left(\prod_{t=1}^{n}\alpha_t < 2^{\bar{\mathsf{R}}}\right) = \int_{\mathcal{A}(\mathsf{r},\mathsf{snr})}f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})d\boldsymbol{\alpha} \quad (148) $$

$$ [\mathbf{E}(\boldsymbol{\omega},\mathsf{snr})]_{u,v} = \begin{cases} \int_1^{\infty}e^{-(\frac{n_{\mathsf{K}}}{\mathsf{snr}})\alpha}{}_0F_1\Big(m-n+1;\omega_u\Big(\frac{n_{\mathsf{K}}}{\mathsf{snr}}\Big)(\alpha-1)\Big)\alpha^{v-(n-k)}d\alpha & 1\le v\le k \\ \frac{\Gamma(m-n+1)}{\Gamma(u-k)\Gamma(m-n+u-k)}\omega_v^{u-k-1} & k<v\le n. \end{cases} \quad (140) $$

where $\mathcal{A}(\mathsf{r},\mathsf{snr}) = \{\boldsymbol{\alpha} \in \mathbb{R}^n | \prod_{t=1}^{n} \alpha_t < 2^{\bar{\mathsf{R}}}, \alpha_1 \geq \cdots \geq \alpha_n \geq 1\}$ and $f_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ denotes the joint pdf of $\boldsymbol{\alpha}$ given in (47) for uncorrelated Rayleigh channels, in (86) for semicorrelated Rayleigh channels, and in (129) for uncorrelated Rician channels.

Observe now that for all $\boldsymbol{\alpha} \in \mathcal{A}(\mathsf{r},\mathsf{snr})$, it holds that $\log_{\mathsf{snr}}(\alpha_t) = 0$ for $t = 1, \ldots, n$, and, hence, we can reuse the derivations in Appendices A.i–A.iii with $k = 0$ to obtain the following.

i. In uncorrelated Rayleigh fading (see Definition 1):

$$\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr}) \sim K_{m,n} A(\bar{\mathsf{R}}) \left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{mn} \tag{149.1}$$

where $K_{m,n}$ is given in (44).

ii. In semicorrelated Rayleigh fading (see Definition 2):

$$\begin{aligned}\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr}) &\sim K_{m,n}(\boldsymbol{\sigma}) A(\bar{\mathsf{R}}) |\mathbf{E}(\boldsymbol{\sigma})| \left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{mn} \\ &= K_{m,n} A(\bar{\mathsf{R}}) |\boldsymbol{\Sigma}|^{-m} \left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{mn} \end{aligned} \tag{149.2}$$

where $K_{m,n}(\boldsymbol{\sigma})$ is given in (84) and matrix $\mathbf{E}(\boldsymbol{\sigma})$ denotes $\mathbf{E}(\boldsymbol{\alpha}_k, \boldsymbol{\sigma})$ defined in (108) for $k = 0$.

iii. In uncorrelated Rician fading (see Definition 3)

$$\begin{aligned}\mathsf{P}_{\mathsf{out}}(\mathsf{r},\mathsf{snr}) &\sim K_{m,n}(\boldsymbol{\omega}) A(\bar{\mathsf{R}}) |\mathbf{E}(\boldsymbol{\omega})| \left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{mn} \\ &= K_{m,n} A(\bar{\mathsf{R}}) \left(\frac{e^{\mathsf{K}}}{\mathsf{K}+1}\right)^{-mn} \left(\frac{n_{\mathsf{T}}}{\mathsf{snr}}\right)^{mn} \end{aligned} \tag{149.3}$$

where $K_{m,n}(\boldsymbol{\omega})$ is given in (127), matrix $\mathbf{E}(\boldsymbol{\omega})$ denotes $\mathbf{E}(\boldsymbol{\alpha}_k, \boldsymbol{\omega})$ defined in (137) for $k = 0$, and the last equality follows from noting that $\mathrm{tr}(\boldsymbol{\Omega}) = \mathsf{K}mn$.

In (149.1)–(149.3), $A(\bar{\mathsf{R}})$ is defined as

$$A(\bar{\mathsf{R}}) = \int_{\mathcal{A}(\mathsf{r},\mathsf{snr})} \prod_{i<j}^{n} (\alpha_j - \alpha_i)^2 \prod_{i=1}^{n} (\alpha_i - 1)^{m-n} d\alpha_n \cdots d\alpha_1 \tag{150}$$

and, similarly to the integral addressed in Appendix A.i.1, it can be calculated as

$$A(\bar{\mathsf{R}}) = \sum_{\boldsymbol{\mu}} \mathrm{sgn}(\boldsymbol{\mu}) A(\boldsymbol{\mu}; \bar{\mathsf{R}}) \tag{151}$$

where the summation over $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)$ is for all permutations of integers $(1, \ldots, n)$ and $A(\boldsymbol{\mu}; \bar{\mathsf{R}})$ is defined as

$$A(\boldsymbol{\mu}; \bar{\mathsf{R}}) = \int_1^{2^{\bar{\mathsf{R}}}} \int_1^{\left(\frac{2^{\bar{\mathsf{R}}}}{x_1}\right)} \cdots \int_1^{\left(\frac{2^{\bar{\mathsf{R}}}}{\prod_{t=1}^{n-1} x_t}\right)} \prod_{t=1}^{n} x_t^{\mu_t + t - 2} (x_t - 1)^{m-n} dx_n \cdots dx_1. \tag{152}$$

Finally, the array gain $\mathsf{a}_j(\bar{\mathsf{R}})$ given in Theorem 2 for channel model j follows from substituting (151) back in (149.j). □

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