Demand-Side Management via Distributed Energy Generation and Storage Optimization

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***Abstract*—Demand-side management, together with the integration of distributed energy generation and storage, are considered increasingly essential elements for implementing the smart grid concept and balancing massive energy production from renewable sources. We focus on a smart grid in which the demand-side comprises traditional users as well as users owning some kind of distributed energy sources and/or energy storage devices. By means of a day-ahead optimization process regulated by an independent central unit, the latter users intend to reduce their monetary energy expense by producing or storing energy rather than just purchasing their energy needs from the grid. In this paper, we formulate the resulting grid optimization problem as a noncooperative game and analyze the existence of optimal strategies. Furthermore, we present a distributed algorithm to be run on the users' smart meters, which provides the optimal production and/or storage strategies, while preserving the privacy of the users and minimizing the required signaling with the central unit. Finally, the proposed day-ahead optimization is tested in a realistic situation.**

***Index Terms*—Demand-side management, distributed energy generation, distributed energy storage, game theory, proximal decomposition algorithm, smart grid.**

I. Introduction

Smart grids have an essential role in the process of transforming the functionalities of the present energy grid in order to provide a user-oriented service and guarantee high security, quality, and economic efficiency of the electricity supply in a market environment. In addition, smart grids are expected to be a key enabler in the transition to a low-carbon energy sector, ensuring the efficient and sustainable use of natural resources [1]. The production from renewable sources as, for instance, wind and photovoltaic units is, however, intermittent in nature and there is often no correlation between the production and the local consumption. Furthermore, since large amounts of variable generation from renewable sources are not fully forecastable, there is an increasing need for flexible, dispatchable, fast-ramping energy generation for balancing variations in load and contingencies such as the loss of transmission or generation assets. Similar problems arise at a market level, since national and local balances between supply and demand are more complicated to manage with high levels of renewable energy generation [2].

In this regard, the concepts of demand-side management (DSM), distributed energy generation (DG), and distributed energy storage (DS) are recognized as main facilitators for the smart grid deployment, since the challenges caused by the integration of renewable energy sources can be minimized when dispatchable DG and DS are incorporated into the demand-side of the electricity network and innovative DSM methods are simultaneously implemented. Indeed, the combination of DG, DS, and DSM techniques results in a system of diverse generation sources supplying energy across the grid to a large set of demand-side users with possibilities for improved energy efficiency, local generation, and controllable loads. Demand-side management refers to the different initiatives intended to modify the time pattern and magnitude of the demand, introducing advanced mechanisms for encouraging the demand-side to participate actively in the network optimization process [3]. Therefore, demand-side users are equipped with a control device, commonly known as smart meter, which communicates with the supply-side and manages their energy demand.

In this paper, we propose a DSM method consisting in a day-ahead optimization process. We focus on those demand-side users, possibly owning DG and DS devices, whose energy consumption is greater than their energy production capabilities. The main objective of these end users is to reduce their monetary expense during the time period of analysis by producing and/or storing energy rather than just purchasing their energy needs from the grid.

Considering the selfish nature of the users, a game theoretical approach is particularly suitable in order to calculate their optimal production and storage strategies. For this reason, we model the day-ahead optimization problem as a noncooperative Nash game and we analyze the existence of the solutions, which correspond to the well-known concept of Nash equilibria, when a practical pricing model (cf. [4], [5]) is applied. Finally, we

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present a distributed and iterative scheme based on the proximal decomposition algorithm that converges to the Nash equilibria with minimum information exchange while safeguarding the privacy of the users.

The rest of the paper is structured as follows. In Section II we describe the overall structure of our smart grid and, specifically, we introduce the production and storage models, as well as the energy cost and pricing model. Section III formulates the optimization problem as a noncooperative game and solves it by means of a specific distributed algorithm. In addition, we derive sufficient conditions for the existence of a solution, as well as for the convergence of the proposed algorithm. In Section IV we show some illustrative numerical results obtained through experimental evaluations. Finally, we provide some concluding remarks in Section V.

II. Smart Grid Model

The goal of this section is to present the overall smart grid model, describe the different types of users belonging to demand-side of the network, and introduce the adopted energy cost and pricing mechanism.

The modern power grid is a complex network comprising several subsystems (power plants, transmission lines, substations, distribution grids, and consumers), which can be conveniently divided into [6]–[8]:

i) *Supply-side*: it includes the utilities (energy producers and providers) and the energy transmission network.
ii) *Central unit*: it is the regulation authority that coordinates the grid optimization process.
iii) *Demand-side*: it incorporates the end users (energy consumers), eventually equipped with DG and/or DS, and the energy distribution network.

Since in this paper we propose a DSM mechanism, we focus our attention only on the demand-side of the smart grid, which is described in detail in Section II-A, whereas the supply-side and the central unit are modeled as plainly as possible.

A. Demand-Side Model

Demand-side users are characterized in the first instance by their individual *per-slot energy consumption profile* $e_n(h)$, defined as the energy needed by user $n \in \mathcal{D}$ to supply his appliances at time-slot h. Accordingly, we also introduce the *energy consumption scheduling vector* $\mathbf{e}_n$, which gathers the energy consumption profiles for the H time-slots in which the time period of analysis is divided, i.e., $\mathbf{e}_n = \left(e_n(h)\right)_{h=1}^{H}$.

Our model classifies the set of all demand-side users $\mathcal{D}$, with cardinality $|\mathcal{D}| = P + N$, into the set of P *passive* users, denoted by $\mathcal{P} \subset \mathcal{D}$, and the set of N *active* users, denoted by $\mathcal{D} \supset \mathcal{N} = \mathcal{D}\backslash\mathcal{P}$. Passive users are basically energy consumers and resemble traditional demand-side users, whereas active users participate in the optimization process, i.e., they react to changes in the cost per unit of energy by modifying their demand. Each active user is connected not only to the bidirectional power distribution grid, but also to a communication infrastructure that enables two-way communication between his smart meter and the central unit (as shown in Fig. 1). The main objective of each active user is to optimize his day-ahead strategy while fulfilling his energy requirements during the time period of analysis, $\mathbf{e}_n$. This strategy depends in the first instance on the equipment owned by user $n \in \mathcal{N}$, e.g., energy sources (see Section II-B) and/or storage devices (see Section II-C), but is also strongly related to the strategy followed by the rest of the active users $\mathcal{N}\backslash\{n\}$ (see Section II-D) and to the aggregate energy consumption of the passive users connected to the grid.

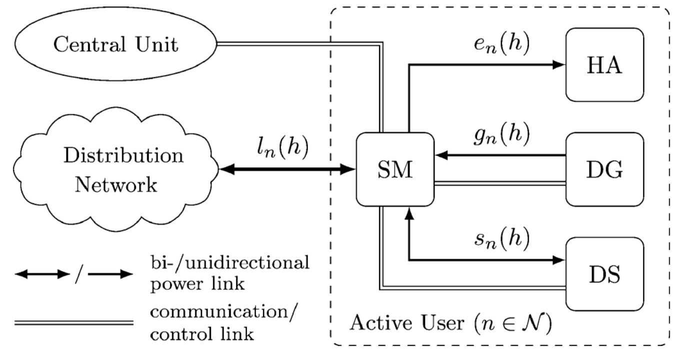

Fig. 1. Connection scheme between the smart grid and one active user consisting of: smart meter (SM), home appliances (HA), distributed energy generation (DG), and distributed energy storage (DS).

Active users include two broad categories: dispatchable energy producers and energy storers. For convenience, we use $\mathcal{G} \subseteq \mathcal{N}$ to denote the subset of users possessing some dispatchable energy source. For users $n \in \mathcal{G}$, $g_n(h) \geq 0$ represents the *per-slot energy production profile* at time-slot h. Likewise, we introduce $\mathcal{S} \subseteq \mathcal{N}$ as the subset of users owning some energy storage device. Users $n \in \mathcal{S}$ are characterized by the *per-slot energy storage profile* $s_n(h)$ at time-slot h: $s_n(h) > 0$ when the storage device is to be charged (i.e., an additional energy consumption), $s_n(h) < 0$ when the storage device is to be discharged (i.e., a reduction of the energy consumption), and $s_n(h) = 0$ when the device is inactive. It is worth remarking that $\mathcal{G} \cup \mathcal{S} = \mathcal{N}$, but we also contemplate active users being both dispatchable producers and storers, i.e., $\mathcal{G} \cap \mathcal{S} \neq \emptyset$.

Finally, we define the *per-slot energy load profile* as

$$l_n(h) = \begin{cases} e_n(h), & \text{if } n \in \mathcal{P} \\ e_n(h) - g_n(h) + s_n(h), & \text{if } n \in \mathcal{N} \end{cases} \tag{1}$$

which expresses the energy flow between user n and the grid at time-slot h, where $l_n(h) > 0$ if the energy flows from the grid to user n and $l_n(h) < 0$ otherwise, as shown in Fig. 1.

B. Energy Production Model

Energy producers can generate energy either to power their own appliances, to charge a storage device, or to sell it to the grid during peak hours. Let us first characterize energy producers depending upon the type of DG they employ [9].

Nondispatchable energy producers $\mathcal{G}_{\mathrm{R}} \subset \mathcal{D}$ using, e.g., renewable resources of intermittent nature such as solar panels or wind turbines. Having only fixed costs, they generate electricity at their maximum available power, which implies no strategy regarding energy production. Consequently, for users $n \in \mathcal{G}_{\mathrm{R}}$, we include nondispatchable generation within the per-slot energy consumption profile $e_n(h)$. Hence, they may have $e_n(h) < 0$

when this energy production is greater than their energy consumption at a given time-slot h. Note that any demand-side user can belong to $\mathcal{G}_{\rm R}$ regardless of his condition of passive or active participant in the optimization process.

Dispatchable energy producers $\mathcal{G} \subseteq \mathcal{N}$ using, e.g., internal combustion engines, gas turbines, or fuel cells. These energy producers, beside fixed costs, have also variable production costs (e.g., the fuel cost) and, therefore, they are interested in optimizing their energy production strategies. In consequence, we introduce the *production cost function* $W_n\big(g_n(h)\big)$, which gives the variable production costs for generating the amount of energy $g_n(h)$ at time-slot h, with $W_n(0)=0$.

Let us now introduce our model for dispatchable energy producers. Let $g_n^{(\max)}$ be the *maximum energy production capability* for user $n \in \mathcal{G}$ over a time-slot. Then, the per-slot energy production profile is bounded as

$$0 \le g_n(h) \le g_n^{(\max)}. \tag{2}$$

For the sake of simplicity, we consider dispatchable energy sources with a fixed instantaneous output power level, which are operated during fractions of a time slot. Hence, $g_n^{(\max)}$ represents the amount of energy produced when user n's energy source operates during 100% of a time-slot. Additionally, the cumulative energy production must satisfy

$$\sum_{h=1}^{H} g_n(h) \le \gamma_n^{(\max)} \tag{3}$$

where $0 < \gamma_n^{(\max)} \le H g_n^{(\max)}$ represents the maximum amount of energy that user $n \in \mathcal{G}$ can generate during the time period of analysis (e.g., to prevent over-usage). Then, introducing $\mathbf{g}_n = \big(g_n(h)\big)_{h=1}^{H}$ as the *energy production scheduling vector*, we define the strategy set $\Omega_{\mathbf{g}_n}$ for dispatchable energy producers $n \in \mathcal{G}$, including constraints (2) and (3), as

$$\Omega_{\mathbf{g}_n} = \left\{ \mathbf{g}_n \in \mathbb{R}_+^H : \mathbf{g}_n \preceq g_n^{(\max)} \mathbf{1}_H, \mathbf{1}_H^{\rm T} \mathbf{g}_n \le \gamma_n^{(\max)} \right\} \tag{4}$$

where the operator $\preceq$ for vectors is defined componentwise, and $\mathbf{1}_H$ is the H-dimensional unit vector.

C. Energy Storage Model

In our model, storage devices (see, e.g., [9], [10] for an overview on storage technologies) of users $n \in \mathcal{S}$ are characterized by: *charging efficiency*, *discharging efficiency*, *leakage rate*, *capacity*, and *maximum charging rate*. If we express the per-slot energy storage profile as $s_n(h) = s_n^{(+)}(h) - s_n^{(-)}(h)$, where $s_n^{(+)}(h), s_n^{(-)}(h) \ge 0$ are the *per-slot energy charging profile* and the *per-slot energy discharging profile*, respectively, the charging and discharging efficiencies $0 < \beta_n^{(+)} \le 1$ and $\beta_n^{(-)} \ge 1$ take into account the conversion losses of the storage device. For instance, if $s_n^{(+)}(h)$ is taken from the grid to be stored on the device, only $\beta_n^{(+)} s_n^{(+)}(h)$ is effectively charged; on the other hand, in order to obtain $s_n^{(-)}(h)$ from the device, $\beta_n^{(-)} s_n^{(-)}(h)$ is to be discharged. The leakage rate $0 < \alpha_n \le 1$ models the decrease in the energy level with the passage of time: if $q_n(h)$ denotes the *charge level* at the end of time-slot h, then it reduces to $\alpha_n q_n(h)$ at the end of time-slot $h+1$. The capacity c_n indicates how much energy the storage device can accumulate. Lastly, the maximum charging rate $s_n^{(\max)}$ is the maximum energy that can be stored during a single time-slot.

Let us introduce the vectors $\mathbf{s}_n(h) = \big(s_n^{(+)}(h), s_n^{(-)}(h)\big)^{\rm T}$ and $\boldsymbol{\beta}_n = (\beta_n^{(+)}, -\beta_n^{(-)})^{\rm T}$: the charge level $q_n(h)$ is given by

$$q_n(h) = \alpha_n q_n(h-1) + \boldsymbol{\beta}_n^{\rm T} \mathbf{s}_n(h) \tag{5}$$

where $q_n(h-1)$ is the charge level at the previous time-slot, which gets reduced by a factor α_n during time-slot h, and $\boldsymbol{\beta}_n^{\rm T} \mathbf{s}_n(h)$ is the energy charged or discharged at h.[1] Since $q_n(h)$ is bounded above by c_n and below by 0, $\mathbf{s}_n(h)$ satisfies

$$-\alpha_n q_n(h-1) \le \boldsymbol{\beta}_n^{\rm T} \mathbf{s}_n(h) \le c_n - \alpha_n q_n(h-1). \tag{6}$$

Moreover, since the maximum charging rate cannot be surpassed, we also have that

$$\boldsymbol{\beta}_n^{\rm T} \mathbf{s}_n(h) \le s_n^{(\max)}. \tag{7}$$

Additionally, it is convenient to include a constraint on the desired charge level at the end of the time period of analysis. The choice of the optimal $q_n(H)$ requires, however, the knowledge of the energy cost at time-slot $H+1$, while the optimization process addressed in this paper only takes in consideration one isolated time period of analysis. In any case, it is reasonable to expect the storage device going through an integer number of cycles of charging to discharging that are opposite to the daily energy demand fluctuation [11]. This implies that the final charge level $q_n(H)$ must be approximately the same as the *initial charge level* $q_n(0)$, i.e., the charge level of user $n \in \mathcal{S}$ at the beginning of time-slot $h=1$. Hence, we have that

$$\big| q_n(H) - q_n(0) \big| \le \epsilon_n \tag{8}$$

where ϵ_n is a sufficiently small positive constant.

Now, we can relate $q_n(h)$ to the initial charge level and to the energy storage profiles at the previous time-slots as

$$q_n(h) = \alpha_n^h q_n(0) + \sum_{t=1}^{h} \alpha_n^{(h-t)} \boldsymbol{\beta}_n^{\rm T} \mathbf{s}_n(t). \tag{9}$$

Given the above expression, and introducing the *energy storage scheduling vector* $\mathbf{s}_n = \Big(\big(s_n^{(+)}(h)\big)_{h=1}^{H}, \big(s_n^{(-)}(h)\big)_{h=1}^{H} \Big)$, we define the strategy set for energy storers $n \in \mathcal{S}$ as $\Omega_{\mathbf{s}_n}$, which combines constraints (6), (7), and (8):

$$\begin{aligned} \Omega_{\mathbf{s}_n} = \Big\{ & \mathbf{s}_n \in \mathbb{R}_+^{2H} : \boldsymbol{\Delta}_{\beta,n} \mathbf{s}_n \preceq s_n^{(\max)} \mathbf{1}_H, \\ & -q_n(0) \mathbf{b}_n \preceq \mathbf{A}_n \boldsymbol{\Delta}_{\beta,n} \mathbf{s}_n \preceq c_n \mathbf{1}_H - q_n(0) \mathbf{b}_n, \\ & \left(1-\alpha_n^H\right) q_n(0) - \epsilon_n \le \mathbf{a}_n^{\rm T} \boldsymbol{\Delta}_{\beta,n} \mathbf{s}_n \le \left(1-\alpha_n^H\right) q_n(0) + \epsilon_n \Big\} \end{aligned} \tag{10}$$

where $\boldsymbol{\Delta}_{\beta,n} = (\beta_n^{(+)} \mathbf{I}_H \; - \beta_n^{(-)} \mathbf{I}_H)$, $\mathbf{A}_n$ is a H-dimensional lower triangular matrix with elements $[\mathbf{A}_n]_{i,j} = \alpha_n^{(i-j)}$, and

[1] Although we do not explicitly impose charging and discharging operations to be mutually exclusive, the optimal storage strategies obtained in Section III satisfy $s_n^{(+)}(h) s_n^{(-)}(h) = 0, \forall h$, whenever $\beta_n^{(+)} < 1$ and $\beta_n^{(-)} > 1$.

TABLE I
DIFFERENT TYPES OF DEMAND-SIDE USERS AND CORRESPONDING STRATEGY SETS

	User subset	Strategy set
$\mathcal{P}$	$\mathcal{P}\backslash\mathcal{G}_{\mathrm{R}}$ $\mathcal{G}_{\mathrm{R}}\backslash(\mathcal{N}\cap\mathcal{G}_{\mathrm{R}})$	No strategy
$\mathcal{N}$	$\mathcal{G}\backslash(\mathcal{G}\cap\mathcal{S})=\mathcal{N}_{\mathcal{G}\backslash\mathcal{S}}$	$\mathbf{g}_n\in\Omega_{\mathbf{g}_n}$
	$\mathcal{S}\backslash(\mathcal{G}\cap\mathcal{S})=\mathcal{N}_{\mathcal{S}\backslash\mathcal{G}}$	$\mathbf{s}_n\in\Omega_{\mathbf{s}_n}$
	$\mathcal{G}\cap\mathcal{S}=\mathcal{N}_{\mathcal{G}\cap\mathcal{S}}$	$(\mathbf{g}_n,\mathbf{s}_n)\in(\Omega_{\mathbf{g}_n}\times\Omega_{\mathbf{s}_n})$

$\mathbf{a}_n$ and $\mathbf{b}_n$ are H-dimensional vectors defined respectively as $[\mathbf{a}_n]_i=\alpha_n^{(H-i)}$ and $[\mathbf{b}_n]_i=\alpha_n^i$.

Finally, it is important to remark that the optimization process analysis and the algorithm presented in Section III hold for any production and storage models resulting in a compact and convex strategy set as the ones in (4) and (10).

After analyzing all possible types of users in the demand-side, we summarize their strategy sets in Table I.

D. Energy Cost and Pricing Model

This section describes the cost model on which depends the price of energy. Let us define the *aggregate per-slot energy load* at time-slot h as

$$0<L(h)=L^{(\mathcal{P})}(h)+\sum_{n\in\mathcal{N}}l_n(h) \tag{11}$$

where $L^{(\mathcal{P})}(h)=\sum_{n\in\mathcal{P}}e_n(h)$ is the aggregate per-slot energy consumption associated with the passive users connected to the grid. Since we are not interested in analyzing overload conditions, throughout the paper we assume that $L(h)<L^{(\max)}$ at each time-slot h, where $L^{(\max)}$ denotes the maximum aggregate energy load that the grid can take before experiencing a blackout.

Let us define the *grid cost function* $C_h\big(L(h)\big)$ indicating the price fixed by the supply-side to provide the aggregate per-slot energy load $L(h)$ at time-slot h. Then, $C_h\big(L(h)\big)\big(l_n(h)/L(h)\big)$ represents the amount of money paid by user n to purchase the energy load $l_n(h)$ from the grid (if $l_n(h)>0$) or received to sell the energy load $l_n(h)$ to the grid (if $l_n(h)<0$) at time-slot h. We adopt the quadratic grid cost function widely used in the smart grid literature (e.g., in [4], [5]):

$$C_h\big(L(h)\big)=K_hL^2(h) \tag{12}$$

with $\{K_h\}_{h=1}^H>0$. In general, the grid coefficients K_h are different at each time-slot h, since the energy production varies along the time period of analysis according to the energy demand and to the availability of intermittent energy sources.

TABLE II
LIST OF IMPORTANT SYMBOLS AND CORRESPONDING DOMAINS

Symbol	Domain	
$l_n(h)$	$l_n(h)\geq 0$	if $n\in\mathcal{P}\backslash\mathcal{G}_{\mathrm{R}}$
	possibly negative	if $n\in\mathcal{N}\cup\mathcal{G}_{\mathrm{R}}$
$e_n(h)$	$e_n(h)\geq 0$	if $n\in\mathcal{N}\backslash\mathcal{G}_{\mathrm{R}}$
	possibly negative	if $n\in\mathcal{G}_{\mathrm{R}}$
$g_n(h)$	$g_n(h)\geq 0$	
$s_n(h)$	$s_n(h)>0$	if charging
	$s_n(h)<0$	if discharging
$L(h)$	$0<L(h)<L^{(\max)}$	

Finally, let $f_n(\mathbf{g}_n,\mathbf{s}_n)$ denote the *cumulative expense* over the time period of analysis, which represents the cumulative monetary expense incurred by user $n\in\mathcal{N}$ for obtaining the desired amount of energy over the time period of analysis:

$$f_n(\mathbf{g}_n,\mathbf{s}_n)=\sum_{h=1}^{H}\Big(K_hL(h)\big(e_n(h)-g_n(h)+s_n(h)\big)+W_n\big(g_n(h)\big)\Big). \tag{13}$$

Note that, in general, the amount of money paid/received by user n to purchase/sell the same amount of energy from/to the grid is different during distinct time-slots due to the fact that the grid cost function and the aggregate per-slot energy load are variable along the day. A summary of the principal variables introduced throughout Section II, along with the corresponding domains, is reported in Table II.

III. DAY-AHEAD OPTIMIZATION PROBLEM

Once defined the overall model, in this section we focus on analyzing the proposed day-ahead optimization problem.

First, the grid energy prices for the time period of analysis, i.e., the grid coefficients $\{K_h\}_{h=1}^H$, are fixed by the supply-side in the day-ahead market-clearing process [6], [8], [11]. Then, each active demand-side user reacts to the prices provided by the central unit through iteratively adjusting his generation and storage strategies $\mathbf{g}_n$ and $\mathbf{s}_n$ and, thus, his day-ahead energy demand $\{l_n(h)\}_{h=1}^H$, with the final objective of minimizing his cumulative expense over the time period of analysis $f_n(\mathbf{g}_n,\mathbf{s}_n)$, given the aggregate energy loads $\{L(h)\}_{h=1}^H$.

By participating in the day-ahead optimization process, demand-side users commit to follow strictly the resulting consumption pattern. Here, we suppose that users know exactly their energy requirements at each time-slot in the time period of analysis in advance and we neglect any real-time fluctuation of such demand (for an overview on real-time pricing mechanisms, we refer to [3], [8]). Additionally, we assume that energy supply follows demand precisely (cf. [7]).

One could consider to solve the previous optimization problem in a centralized fashion, with the central unit imposing every active user how much energy he must produce, charge, and discharge at each time-slot. However, this represents a quite invasive solution, since it requires each user to provide detailed information about his energy production and/or storage capabilities. Indeed, these privacy issues may discourage the demand-side users to subscribe to the optimization process. Besides, a centralized approach is not scalable and cannot account for an unpredictably increasing number of participants. In consequence, we are interested instead in a *fully distributed* solution and, hence, a game theoretical approach is remarkably suitable to accommodate our optimization problem (see [12] for an overview on game theory applied on smart grids).

A. Game Theoretical Formulation

Game theory is a field of applied mathematics that describes and analyzes scenarios with interactive decisions [13]. Here, we model the optimization process as a noncooperative Nash game. Each active user is a player who competes against the others by choosing, given the per-slot aggregate energy loads at each iteration, the production and storage strategies $\mathbf{g}_n$ and $\mathbf{s}_n$ that minimize his payoff function, i.e., his cumulative expense over the time period of analysis. Since these individual strategies impact the grid energy price of all users, this leads to a coupled problem where the desired solution is an equilibrium point where all users are unilaterally satisfied.

First, let us define the strategy vector and the corresponding per-slot strategy profile of a generic user $n \in \mathcal{N}$ as

$$\mathbf{x}_n = (\mathbf{g}_n, \mathbf{s}_n) \tag{14}$$

$$\mathbf{x}_n(h) = \big(g_n(h), \mathbf{s}_n(h)\big)^{\mathrm{T}}. \tag{15}$$

For convenience, we divide the users participating actively in the optimization in three main groups (see Table I for details):

i) *Dispatchable energy producers*: $\mathcal{N}_{\mathcal{G}\setminus\mathcal{S}} = \mathcal{G}\setminus(\mathcal{G}\cap\mathcal{S})$, for whom $\mathbf{g}_n \in \Omega_{\mathbf{g}_n}$ and $\mathbf{s}_n = \mathbf{0}$.

ii) *Energy storers*: $\mathcal{N}_{\mathcal{S}\setminus\mathcal{G}} = \mathcal{S}\setminus(\mathcal{G}\cap\mathcal{S})$, for whom $\mathbf{s}_n \in \Omega_{\mathbf{s}_n}$ and $\mathbf{g}_n = \mathbf{0}$.

iii) *Dispatchable energy producers-storers*: $\mathcal{N}_{\mathcal{G}\cap\mathcal{S}} = \mathcal{G}\cap\mathcal{S}$, for whom $\mathbf{g}_n \in \Omega_{\mathbf{g}_n}$ and $\mathbf{s}_n \in \Omega_{\mathbf{s}_n}$.

Taking into account the production and storage feasible sets $\Omega_{\mathbf{g}_n}$ and $\Omega_{\mathbf{s}_n}$ introduced in Sections II-B and II-C, respectively, we can now characterize the strategy set of a generic user $n \in \mathcal{N}$ as

$$\Omega_{\mathbf{x}_n} = \begin{cases} \mathbf{g}_n \in \Omega_{\mathbf{g}_n},\ \mathbf{s}_n = \mathbf{0}, & \text{if } n \in \mathcal{N}_{\mathcal{G}\setminus\mathcal{S}} \\ \mathbf{g}_n = \mathbf{0},\ \mathbf{s}_n \in \Omega_{\mathbf{s}_n}, & \text{if } n \in \mathcal{N}_{\mathcal{S}\setminus\mathcal{G}} \\ \mathbf{g}_n \in \Omega_{\mathbf{g}_n},\ \mathbf{s}_n \in \Omega_{\mathbf{s}_n}, & \text{if } n \in \mathcal{N}_{\mathcal{G}\cap\mathcal{S}} \end{cases}. \tag{16}$$

It is worth pointing out that the strategy sets $\Omega_{\mathbf{x}_n}$ are decoupled. Bearing in mind the pricing model given in (13), the payoff function of user n is given by

$$f_n\big(\mathbf{x}_n, \mathbf{l}_{-n}\big) = \sum_{h=1}^{H} K_h\big(l_{-n}(h) + e_n(h) + \boldsymbol{\delta}^{\mathrm{T}}\mathbf{x}_n(h)\big) \times\big(e_n(h) + \boldsymbol{\delta}^{\mathrm{T}}\mathbf{x}_n(h)\big) + \sum_{h=1}^{H} W_n\big(\boldsymbol{\delta}_g^{\mathrm{T}}\mathbf{x}_n(h)\big) \tag{17}$$

where $\mathbf{l}_{-n} = \big(l_{-n}(h)\big)_{h=1}^{H}$, with $l_{-n}(h) = L^{(\mathcal{P})}(h) + \sum_{m\in\mathcal{N}\setminus\{n\}} l_m(h)$ being the aggregate per-slot energy load of the other players $m \in \mathcal{N}\setminus\{n\}$ at time-slot h, and where we have introduced the auxiliary vectors $\boldsymbol{\delta} = (-1, 1, -1)^{\mathrm{T}}$ and $\boldsymbol{\delta}_g = (1, 0, 0)^{\mathrm{T}}$.

We can now formally define the game among the active users as $\mathscr{G} = \langle\Omega_{\mathbf{x}}, \mathbf{f}\rangle$, with $\Omega_{\mathbf{x}} = \prod_{n=1}^{N}\Omega_{\mathbf{x}_n}$ and $\mathbf{f} = \big(f_n(\mathbf{x}_n, \mathbf{l}_{-n})\big)_{n=1}^{N}$. The final objective of each player $n \in \mathcal{N}$ is to choose his own strategy $\mathbf{x}_n \in \Omega_{\mathbf{x}_n}$ in order to minimize his payoff function $f_n(\mathbf{x}_n, \mathbf{l}_{-n})$, given the aggregate energy load vector of the other players $\mathbf{l}_{-n}$:

$$\begin{array}{ll} \min\limits_{\mathbf{x}_n} & f_n(\mathbf{x}_n, \mathbf{l}_{-n}) \\ \text{s.t.} & \mathbf{x}_n \in \Omega_{\mathbf{x}_n} \end{array} \qquad \forall n \in \mathcal{N}. \tag{18}$$

Then, the solution of the game $\mathscr{G} = \langle\Omega_{\mathbf{x}}, \mathbf{f}\rangle$ corresponds to the well-known concept of Nash equilibrium, which is a feasible strategy profile $\mathbf{x}^\star = \big(\mathbf{x}_n^\star\big)_{n=1}^{N}$ with the property that no single player n can profitably deviate from his strategy $\mathbf{x}_n^\star$, if all other players act according to their optimal strategies [13].

B. Analysis of Nash Equilibria

The objective of this section is to study the existence of the Nash equilibria of the game $\mathscr{G} = \langle\Omega_{\mathbf{x}}, \mathbf{f}\rangle$ in (18), with $\Omega_{\mathbf{x}_n}$ given in (16). Sufficient conditions to guarantee the existence of such Nash equilibria are derived in the next theorem.

Theorem 1: Given the game $\mathscr{G} = \langle\Omega_{\mathbf{x}}, \mathbf{f}\rangle$ in (18), suppose that the production cost function $W_n(x)$ is convex in $0 \le x \le g_n^{(\max)}$, $\forall n \in \mathcal{G}$. Then, the following hold:

a) The game has a nonempty and compact solution set.

b) The payoff function of each player is constant over the solution set of the game, i.e., all Nash equilibria yield the same values of the payoff functions.

Proof (a): The game $\mathscr{G} = \langle\Omega_{\mathbf{x}}, \mathbf{f}\rangle$ has a nonempty and compact solution set if [14, Th. 4.1(a)]: i) the individual strategy sets $\Omega_{\mathbf{x}_n}$ in (16) are compact and convex; ii) the payoff functions $f_n(\mathbf{x}_n, \mathbf{l}_{-n})$ in (17) are convex for any feasible $\mathbf{l}_{-n}$. The first condition is immediately satisfied since the sets $\Omega_{\mathbf{x}_n}$, i.e., (4) and (10), are defined as sets of linear inequalities, i.e., polyhedrons [15, Sec. 2.2.4], and they thus form compact and convex sets. Hence, we only need to verify the second condition. The payoff function $f_n(\mathbf{x}_n, \mathbf{l}_{-n})$ is convex if its Hessian matrix $\mathbf{H}(f_n)$, with block elements

$$\begin{array}{l} \nabla^2_{\mathbf{x}_n(h_1)\mathbf{x}_n(h_1)} f_n(\mathbf{x}_n, \mathbf{l}_{-n}) = 2K_h\boldsymbol{\delta}\boldsymbol{\delta}^{\mathrm{T}} + \boldsymbol{\delta}_g\boldsymbol{\delta}_g^{\mathrm{T}} W_n''\big(\boldsymbol{\delta}_g^{\mathrm{T}}\mathbf{x}_n(h)\big) \\ \nabla^2_{\mathbf{x}_n(h_1)\mathbf{x}_n(h_2)} f_n(\mathbf{x}_n, \mathbf{l}_{-n}) = \mathbf{0}_3, \quad h_1 \neq h_2 \end{array} \tag{19}$$

with $\mathbf{0}_a$ denoting the a-dimensional zero matrix, is positive semidefinite. Since $\{K_h\}_{h=1}^{H} > 0$ and the matrix $\boldsymbol{\delta}\boldsymbol{\delta}^{\mathrm{T}}$ has nonnegative eigenvalues, $\mathbf{H}(f_n)$ is guaranteed to be positive semidefinite if $W_n(x)$ is convex, i.e., if $W_n''(x) \ge 0$. Nevertheless, $f_n(\mathbf{x}_n, \mathbf{l}_{-n})$ must be convex $\forall n \in \mathcal{N}$ and, therefore, this constraint must hold $\forall n \in \mathcal{G}$. □

Proof (b): Although the Nash equilibrium is not unique, all Nash equilibria happen to have the same quality. In fact, consider a generic user $n \in \mathcal{N}_{\mathcal{G}\cap\mathcal{S}}$: given two optimal strategy vectors $\mathbf{x}_{1,n}^\star \neq \mathbf{x}_{2,n}^\star$, with $\mathbf{x}_{1,n}^\star = (\mathbf{g}_{1,n}, \mathbf{s}_{1,n})$ and $\mathbf{x}_{2,n}^\star =$

$(\mathbf{g}_{2,n}, \mathbf{s}_{2,n})$, we have that $f_n(\mathbf{x}^\star_{1,n}, \mathbf{l}_{-n}) = f_n(\mathbf{x}^\star_{2,n}, \mathbf{l}_{-n})$ if the following $H+2$ conditions hold:

$$\sum_{h=1}^{H} W_n\big(g_{1,n}(h)\big) = \sum_{h=1}^{H} W_n\big(g_{2,n}(h)\big) \tag{20}$$

$$s_{1,n}(h) - g_{1,n}(h) = s_{2,n}(h) - g_{2,n}(h), \qquad \forall h \tag{21}$$

$$\sum_{h=1}^{H} \alpha_n^{(H-h)} \boldsymbol{\beta}_n^{\mathrm{T}} \mathbf{s}_{1,n}(h) = \sum_{h=1}^{H} \alpha_n^{(H-h)} \boldsymbol{\beta}_n^{\mathrm{T}} \mathbf{s}_{2,n}(h) \tag{22}$$

where the equality in (22) comes from the constraint in (8). Hence, being $\mathbf{x}_n \in \mathbb{R}^{3H}$ and $H > 1$, it follows that user $n \in \mathcal{N}_{\mathcal{G}\cap\mathcal{S}}$ can choose among infinitely many optimal strategy vectors $\mathbf{x}^\star_n$, each of them giving the same value of $f_n(\mathbf{x}^\star_n, \mathbf{l}_{-n})$. Furthermore, since $\mathbf{x}^\star_n$ produces the same $\{l^\star_n(h)\}_{h=1}^{H}$, $\forall n \in \mathcal{N}$, the aggregate loads $\{L^\star(h)\}_{h=1}^{H}$, with $L^\star(h) = L^{\mathcal{P}} + \sum_{n\in\mathcal{N}}^{H} l^\star_n(h)$, are not affected by the multiplicity of the Nash equilibria. Hence, any $\mathbf{x}^\star = (\mathbf{x}^\star_n)_{n=1}^{N}$ yields the same values of the payoff functions $\{f_n(\mathbf{x}^\star_n, \mathbf{l}_{-n})\}_{n\in\mathcal{N}}$. □

Remark 1.1: The convexity of $W_n(\cdot)$ required by Theorem 1 simply implies that the production cost function does not tend to saturate as the per-slot energy production profile increases, which is a very reasonable assumption.

C. Computation of Nash Equilibria

Once we have established the conditions under which the Nash equilibria of the game $\mathscr{G} = \langle \Omega_{\mathbf{x}}, \mathbf{f}\rangle$ exist, we are interested in obtaining a suitable distributed algorithm to compute one of these equilibria with minimum information exchange among the users. Since in a Nash game every player tries to minimize his own objective function, a natural approach is to consider an *iterative* algorithm where, at every iteration i, each individual user n updates his strategy by minimizing his payoff function

$$f_n(\mathbf{x}_n, \mathbf{l}_{-n}^{(i)}) = \sum_{h=1}^{H} \Big(K_h\big(l_{-n}^{(i)}(h) + e_n(h) + \boldsymbol{\delta}^{\mathrm{T}}\mathbf{x}_n(h)\big) \times \big(e_n(h) + \boldsymbol{\delta}^{\mathrm{T}}\mathbf{x}_n(h)\big) + W_n\big(\boldsymbol{\delta}_g^{\mathrm{T}}\mathbf{x}_n(h)\big)\Big) \tag{23}$$

referring to the value of the aggregate energy load vector of the other users calculated at the iteration i, i.e., $\mathbf{l}_{-n}^{(i)} = \big(l_{-n}^{(i)}(h)\big)_{h=1}^{H}$, with $l_{-n}^{(i)} = L^{(\mathcal{P})}(h) + \sum_{m\in\mathcal{N}\setminus\{n\}} l_m^{(i)}(h)$.

Recall that, in the game (18), the coupling between users lies at the level of the payoff functions $f_n(\mathbf{x}_n, \mathbf{l}_{-n})$, whereas the feasible sets $\Omega_{\mathbf{x}_n}$ are decoupled. Distributed algorithms based on the individual best-responses of the players [14, Alg. 4.1] represent an extremely flexible and easy-to-implement solution. The conditions ensuring the convergence of these algorithms, however, may not be easy to fulfill: in fact, following [14, Th. 4.2], it is not difficult to show that their convergence cannot be guaranteed in our case if the users are allowed to simultaneously adopt production and storage strategies.

To overcome this issue, we consider a distributed algorithm based on the proximal decomposition [14, Alg. 4.2], which is guaranteed to converge under milder conditions on the system specifications and some additional constraints on the parameters of the algorithm that we provide next in Theorem 2. Given $\mathbf{x}^{(i)} = \big(\mathbf{x}_n^{(i)}\big)_{n=1}^{N} \in \Omega_{\mathbf{x}}$, consider the regularized game

$$\begin{array}{ll} \displaystyle\min_{\mathbf{x}_n} & f_n(\mathbf{x}_n, \mathbf{l}_{-n}) + \dfrac{\tau}{2}\|\mathbf{x}_n - \mathbf{x}_n^{(i)}\|^2 \\ \text{s.t.} & \mathbf{x}_n \in \Omega_{\mathbf{x}_n} \end{array} \qquad \forall n \in \mathcal{N}. \tag{24}$$

which, for a sufficiently large regularization parameter $\tau > 0$, has a unique solution that can be computed in a distributed way using the best-response algorithm [14, Cor. 4.1]. Furthermore, the sequence generated by a proper averaging of the solution of the regularized game (24) and $\mathbf{x}^{(i)}$ converges to a solution of the game (18) (we refer to [14, Ch. 4.2.4.2] for details). This idea is formalized in Algorithm 1.

Algorithm 1 Proximal Decomposition Algorithm

`Data :` Set i $= 0$ and the initial centroid $(\bar{\mathbf{x}}_n^{(0)})_{n=1}^{N} = \mathbf{0}$.
Given $\{K_h\}_{h=1}^{H}$, any feasible starting point $\mathbf{x}^{(0)} = (\mathbf{x}_n^{(0)})_{n=1}^{N}$, and $\tau > 0$:

`(S.1): If` a suitable termination criterion is satisfied: `STOP`.

`(S.2): For` $n \in \mathcal{N}$, each user computes $\mathbf{x}_n^{(i+1)}$ as

$$\mathbf{x}_n^{(i+1)} \in \underset{\mathbf{x}_n\in\Omega_{\mathbf{x}_n}}{\operatorname{argmin}} \left\{ f_n(\mathbf{x}_n, \mathbf{l}_{-n}^{(i)}) + \frac{\tau}{2}\|\mathbf{x}_n - \bar{\mathbf{x}}_n\|^2 \right\} \tag{25}$$

`End`

`(S.3): If` the NE has been reached, each user $n \in \mathcal{N}$ updates his centroid: $\bar{\mathbf{x}}_n = \mathbf{x}_n^{(i+1)}$.

`(S.4):` $i \leftarrow i+1$; `Go to (S.1).`

Next theorem provides sufficient conditions for the convergence of Algorithm 1 to a solution of the game $\mathscr{G} = \langle \Omega_{\mathbf{x}}, \mathbf{f}\rangle$.

Theorem 2: Given the game $\mathscr{G} = \langle \Omega_{\mathbf{x}}, \mathbf{f}\rangle$ in (18), suppose that the following conditions hold:

a) The production cost function $W_n(x)$ is convex in $0 \le x \le g_n^{(\max)}$, $\forall n \in \mathcal{G}$;
b) The regularization parameter τ satisfies

$$\tau > 3(N-1)\max_h K_h. \tag{26}$$

Then, any sequence $\{\mathbf{x}_n^{(i)}\}_{i=1}^{\infty}$ generated by Algorithm 1 converges to a Nash equilibrium of the game.

Proof: Algorithm 1 is an instance of the proximal decomposition algorithm, which is presented in [14, Alg. 4.2] for the variational inequality problem. Next, we rewrite the convergence conditions exploiting the equivalence between game theory and variational inequality (see [14, Ch. 4.2] for details). Given $f_n(\mathbf{x}_n, \mathbf{l}_{-n})$ defined as in (17), Algorithm 1 converges if the following two conditions are satisfied: i) the Jacobian $\mathbf{J}(\mathbf{x})$

of $\left(\nabla_{\mathbf{x}_n} f_n(\mathbf{x}_n, \mathbf{l}_{-n})\right)_{n=1}^{N}$ is positive semidefinite $\forall \mathbf{x} \in \Omega_{\mathbf{x}}$ [14, Th. 4.3]; ii) the $N \times N$ matrix $\boldsymbol{\Upsilon}_{\mathrm{F},\tau} = \boldsymbol{\Upsilon}_{\mathrm{F}} + \tau \mathbf{I}_N$, with

$$[\boldsymbol{\Upsilon}_{\mathrm{F}}]_{nm} = \begin{cases} \upsilon_n^{(\min)}, & \text{if } n = m \\ -\upsilon_{nm}^{(\max)}, & \text{if } n \neq m \end{cases} \tag{27}$$

is a P-matrix [14, Cor. 4.1], where we have introduced

$$\upsilon_n^{(\min)} = \min_{\mathbf{x} \in \Omega_{\mathbf{x}}} \lambda_{\min}\left\{\mathbf{J}_{nn}(\mathbf{x})\right\} \tag{28}$$

$$\upsilon_{nm}^{(\max)} = \max_{\mathbf{x} \in \Omega_{\mathbf{x}}} \left\|\mathbf{J}_{nm}(\mathbf{x})\right\| \tag{29}$$

with $\lambda_{\min}\{\cdot\}$ denoting the smallest eigenvalue of the matrix argument. We can write the block elements of $\mathbf{J}(\mathbf{x})$ as

$$\mathbf{J}_{nn}(\mathbf{x}) = 2\boldsymbol{\Delta}^{\mathrm{T}}\mathbf{K}\boldsymbol{\Delta} + \boldsymbol{\Delta}_g^{\mathrm{T}}\mathbf{D}_{W_n''}(\mathbf{x}_n)\boldsymbol{\Delta}_g \tag{30}$$

$$\mathbf{J}_{nm}(\mathbf{x}) = \boldsymbol{\Delta}^{\mathrm{T}}\mathbf{K}\boldsymbol{\Delta}, \qquad n \neq m \tag{31}$$

where we have introduced the H-dimensional diagonal matrices $\mathbf{D}_{W_n''}(\mathbf{x}_n) = \mathrm{Diag}\left(W_n''(\boldsymbol{\delta}_g^{\mathrm{T}}\mathbf{x}_n(1)), \ldots, W_n''(\boldsymbol{\delta}_g^{\mathrm{T}}\mathbf{x}_n(H))\right)$ and $\mathbf{K} = \mathrm{Diag}(K_1, \ldots, K_H)$, and the auxiliary matrices $\boldsymbol{\Delta} = (-\mathbf{I}_H \ \mathbf{I}_H \ -\mathbf{I}_H)$ and $\boldsymbol{\Delta}_g = (-\mathbf{I}_H \ \mathbf{0}_H \ \mathbf{0}_H)$.

We show next that conditions a) and b) in Theorem 2 imply i) and ii), respectively. Since $\left\{K_h\right\}_{h=1}^{H} > 0$, the terms in (31) are positive semidefinite. On the other hand, the positive semidefiniteness of the diagonal terms in (30), and thereby the inequality $\mathbf{J}(\mathbf{x}) \succeq 0$, is also guaranteed if $W_n''(x) \geq 0, \forall n \in \mathcal{G}$, as required by Theorem 2(a). On the other hand, considering $\mathbf{J}_{nn}(\mathbf{x})$ and $\mathbf{J}_{nm}(\mathbf{x})$ in (30)–(31), we have that $\upsilon_n^{(\min)} \geq 0$ and $\upsilon_{nm}^{(\max)} \leq 3 \max_h K_h$. Then, it follows from [14, Prop 4.3] that, if τ is chosen as in Theorem 2(b), the matrix $\boldsymbol{\Upsilon}_{\mathrm{F},\tau}$ is a P-matrix, which completes the proof. □

Finally, we can describe the proposed day-ahead optimization as follows. At the beginning of the optimization process, τ is computed as in Theorem 2(b) and broadcast to each user $n \in \mathcal{N}$, together with the grid coefficients $\left\{K_h\right\}_{h=1}^{H}$. Then, at each iteration i, the central unit broadcasts a synchronization signal and all users update their centroid $\bar{\mathbf{x}}_n$ simultaneously. Within each iteration, each active user computes his strategy by solving his own optimization problem in (25) referring to the aggregate energy load vector of the other users $\mathbf{l}_{-n}^{(i-1)}$, until equilibrium in the inner loop in (S.2) is reached. Indeed, user n receives the aggregate energy loads $\left\{L(h)\right\}_{h=1}^{H}$, which are calculated by the central unit summing up the individual demands provided by all users, and he obtains $\mathbf{l}_{-n}^{(i-1)}$ by subtracting his own energy loads at the previous iteration $i-1$. Lastly, as indicated in (S.1) of Algorithm 1, the central unit finalizes the whole process when some termination criterion is met as, for instance, when the relative modification in the energy loads of all users between two consecutive iterations is sufficiently small: $\|\mathbf{l}^{(i)} - \mathbf{l}^{(i-1)}\|_2 / \|\mathbf{l}^{(i)}\|_2 \leq \varepsilon$, where $\mathbf{l}^{(i)} = \left(\left(l_n^{(i)}(h)\right)_{h=1}^{H}\right)_{n=1}^{N}$. Note that the individual strategies are not revealed among the users in any case, and only the aggregate energy loads, which are determined at the central unit adding the individual day-ahead energy demands, are communicated by the central unit to each active user.

IV. Simulation Results

In this section, we provide some numerical results that illustrate the performance of the proposed day-ahead DSM mechanism based on the proximal decomposition algorithm described in Algorithm 1. Two different cases of analysis are examined: Case 1 delineates the overall results of our optimization process, examines the convergence of Algorithm 1, and compares the benefits achieved by the different types of active users, showing that they all have substantially reduced their monetary expense by adopting distributed energy generation and/or storage; Case 2 evaluates the day-ahead optimization process with different percentages of active users.

We test the performance of Algorithm 1 within a smart grid of 1000 demand-side users, considering a time period of analysis of one day divided in $H = 24$ time-slots of one hour each. Each demand-side user $n \in \mathcal{D}$ has a random energy consumption curve with daily average of $\sum_{h=1}^{24} e_n(h) = 12$ kWh [16], where higher consumption occurs more likely during day-time hours, i.e., from 08:00 to 24:00, than during night-time hours, i.e., from 00:00 to 08:00, reaching its peak between 17:00 and 23:00. Setting $L^{(\max)} = N \times 3$ kWh, we use the quadratic grid cost function introduced in (12), with

$$C_h\left(L(h)\right) = K_h L^2(h) = \begin{cases} K_{\text{night}} L^2(h), & \text{for } h = 1, \ldots, 8 \\ K_{\text{day}} L^2(h), & \text{for } h = 9, \ldots, 24 \end{cases} \tag{32}$$

where $K_{\text{day}} = 1.5 K_{\text{night}}$ as in [5], and whose values are chosen in order to obtain an initial average price per kWh of 0.1412 £/kWh [17]. Besides, we suppose that dispatchable energy producers $n \in \mathcal{G}$ have a linear production cost function, resembling that of a combustion engine (e.g., a biomass generator [18]) working in the linear region, given by

$$W_n(x) = \eta_n x, \quad \eta_n > 0, \ \forall n \in \mathcal{G}. \tag{33}$$

For the sake of simplicity, we assume that all dispatchable energy producers adopt a generator characterized by the linear production cost function in (33), with $\eta_n = 0.039$ £/kWh [19]. Furthermore, we arbitrarily set $g_n^{(\max)} = 0.4$ kW and $\gamma_n^{(\max)} = 0.8 g_n^{(\max)} \times 24$ h, $\forall n \in \mathcal{G}$. Likewise, we suppose that all energy storers use the same type of storage device, e.g., a lithium-ion battery [20] with $\alpha_n = \sqrt[24]{0.9}$ (which corresponds to a leakage rate of 0.9 over the 24 hours), $\beta_n^{(+)} = 0.9$, $\beta_n^{(-)} = 1.1$, $c_n = 4$ kWh (same value used in [11]), $s_n^{(\max)} = 0.125 c_n/\text{h}$, $q_n(0) = 0.25 c_n$, and $\epsilon_n = 0$, $\forall n \in \mathcal{S}$.

A. Case 1: Overall Performance

In this first case of analysis, we consider a smart grid comprising $N = 180$ active users, where $n = \{1, \ldots, 60\} \in \mathcal{N}_{\mathcal{G} \cap \mathcal{S}}$,

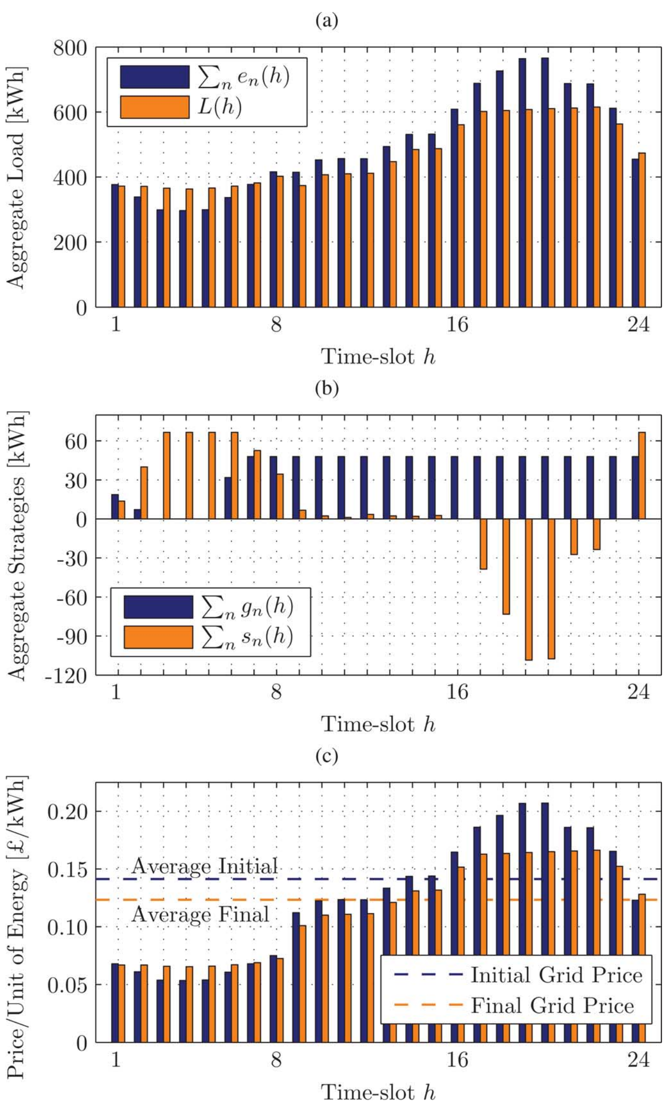

Fig. 2. Case 1: (a) Aggregate per-slot initial consumption and energy loads resulting from Algorithm 1. (b) Aggregate per-slot energy production and storage. (c) Initial and final grid prices per unit of energy.

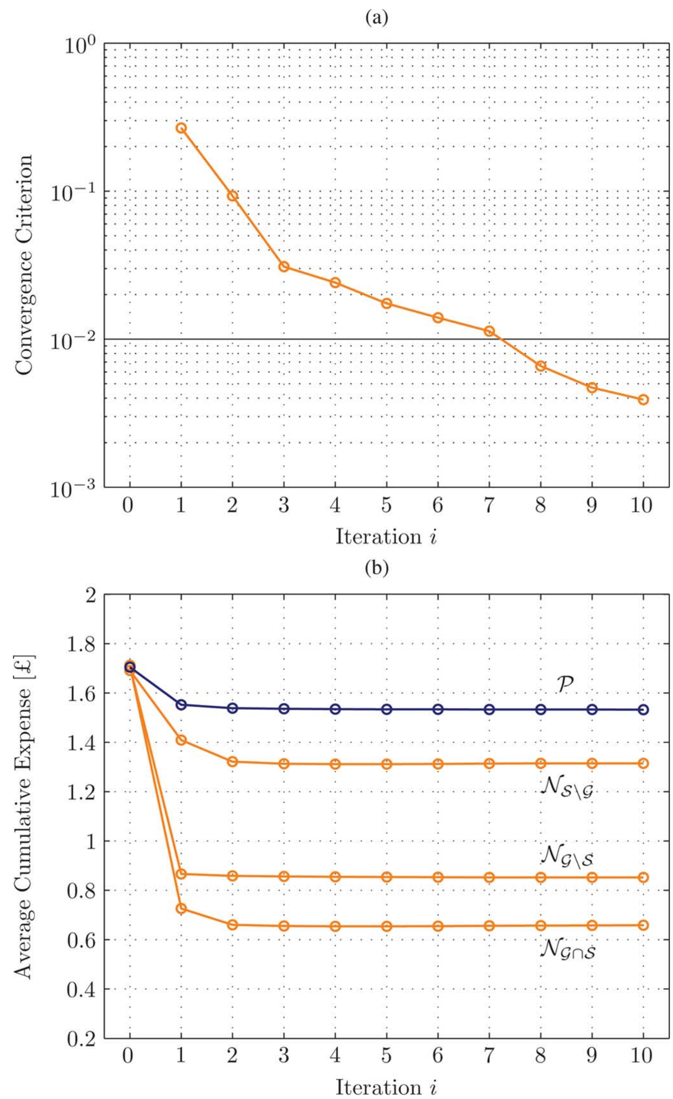

Fig. 3. Case 1: (a) Convergence of Algorithm 1 with termination criterion $\|\mathbf{l}^{(i)} - \mathbf{l}^{(i-1)}\|_2/\|\mathbf{l}^{(i)}\|_2 \leq 10^{-2}$. (b) Average cumulative expense over the time period of analysis for each subset of active users, at each iteration i.

$n = \{61, \ldots, 120\} \in \mathcal{N}_{\mathcal{S}\setminus\mathcal{G}}$, $n = \{121, \ldots, 180\} \in \mathcal{N}_{\mathcal{G}\setminus\mathcal{S}}$, respectively, and $P = 820$ passive users $n = \{181, \ldots, 1000\} \in \mathcal{P}$; this corresponds to having 18% of active users equally distributed among dispatchable energy producers, energy storers, and dispatchable energy producers-storers. Moreover, we arbitrarily set the daily energy consumption for each demand-side user ranging between 8 kWh and 16 kWh. Fig. 2(a) shows the aggregate energy consumption $\sum_{n\in\mathcal{D}} e_n(h)$ together with the aggregate load $L(h)$ at each hour h resulting from Algorithm 1, while Fig. 2(b) delineates the aggregate per-slot energy production $\sum_{n\in\mathcal{G}} g_n(h)$ and storage $\sum_{n\in\mathcal{S}} s_n(h)$ at each hour h. As expected, energy storers charge their battery at the valley of the energy cost, substantially flattening the demand curve. Contrarily, they discharge it at peak hours, shaving off the peak of the load. Likewise, dispatchable producers generate little energy during night-time hours, when they rather purchase it from the grid.

The average grid price per kWh reduces to 0.1234 £/kWh (i.e., 12.6% less). Considering the individual energy production cost for users $n \in \mathcal{G}$, the overall price further decreases to 0.1171 £/kWh. The comparison between the initial and the final grid price at each hour h is illustrated in Fig. 2(c). Moreover, the total expense $\sum_{n\in\mathcal{D}} f_n(\mathbf{g}_n, \mathbf{s}_n)$ reduces from 1704 £ to 1426 £ (i.e., 16.3% less). Finally, the peak-to-average ratio (PAR), calculated as $\mathrm{PAR} = \big(H \max_h L(h)\big)/\big(\sum_{h=1}^{H} L(h)\big)$ decreases from 1.5223 to 1.3129 (i.e., 13.8% less) resulting in a generally flattened demand curve.

Fig. 3(a) plots the termination criterion $\|\mathbf{l}^{(i)} - \mathbf{l}^{(i-1)}\|_2/\|\mathbf{l}^{(i)}\|_2 \leq 10^{-2}$ that finalizes Algorithm 1, over the first 10 iterations. With the above setup, convergence is reached after $i = 8$ iterations. However, Fig. 3(b) shows that active users approximately converge to their final value of the payoff function $f_n(\mathbf{g}_n, \mathbf{s}_n)$ after just $i = 2$ iterations, although they keep adjusting their strategies until the termination criterion is met. Furthermore, from Fig. 3(b) it is straightforward to conclude that active users with more degrees of freedom (i.e.,

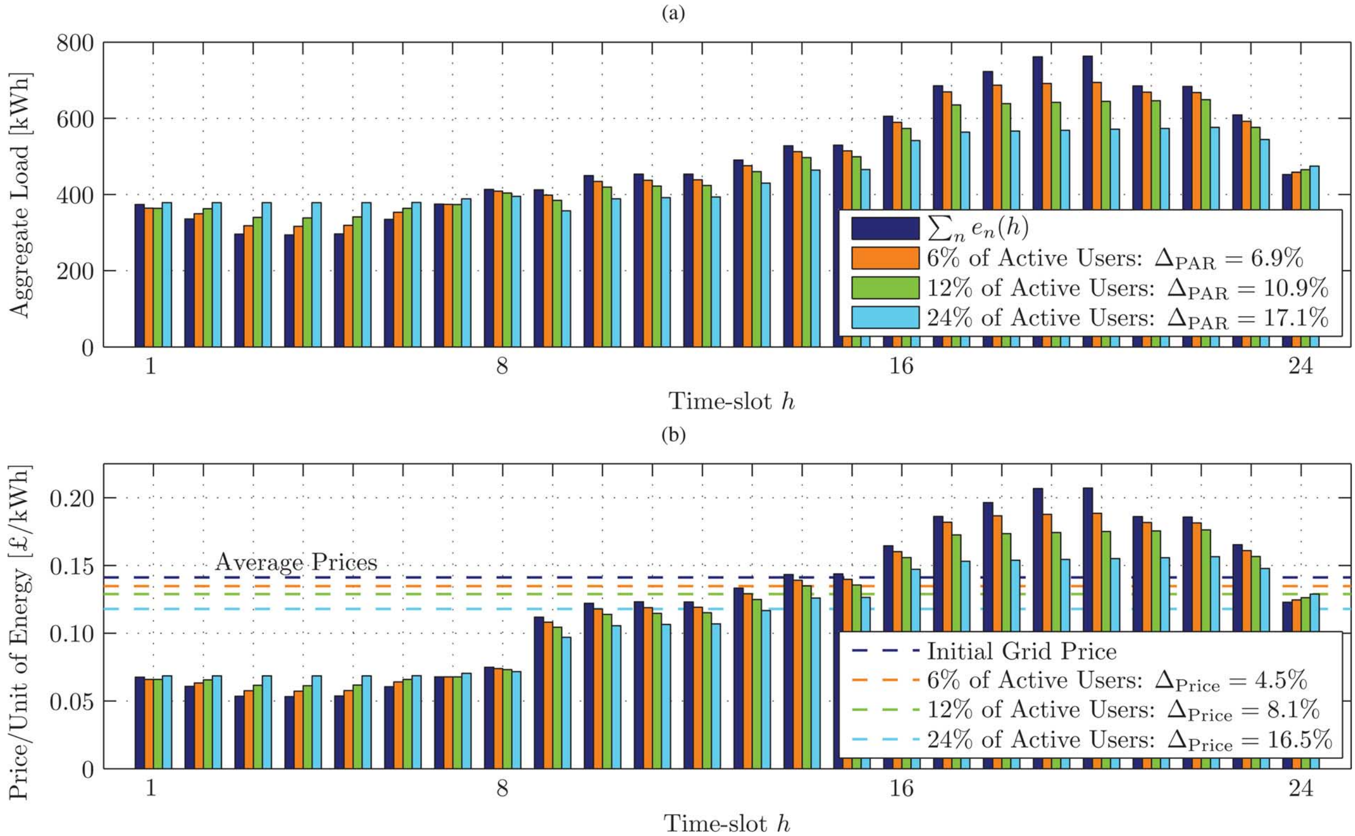

Fig. 4. Case 2: (a) Aggregate per-slot energy loads (Δ_{PAR} = decrease in the PAR); (b) Initial and final grid prices (Δ_{Price} = decrease in the grid price).

both storage and production equipment) obtain better saving percentages, although the employment of DG and DS benefits all users in the network. In particular, the average savings obtained for each subset of active users are: 1.0539 £ (i.e., 61.4% less) for $n \in \mathcal{N}_{\mathcal{G}\cap\mathcal{S}}$, 0.8562 £ (i.e., 50.1% less) for $n \in \mathcal{N}_{\mathcal{G}\setminus\mathcal{S}}$, and 0.3766 £ (i.e., 22.2% less) for $n \in \mathcal{N}_{\mathcal{S}\setminus\mathcal{G}}$. On the other hand, passive users $n \in \mathcal{P}$ save on average 0.1718 £ (i.e., 10.1% less) each. Evidently, the saving for users $n \in \mathcal{N}$ is greater than for users $n \in \mathcal{P}$, i.e., all demand-side users are incentivized to directly adopt DG and/or DS. Moreover, using both dispatchable energy sources and storage devices allows users to further decrease their individual cumulative expenses.

B. Case 2: Comparison Between Different Percentages of Active Users

In this second case of analysis, we compare the benefits given by the day-ahead optimization process addressed in this paper with different percentages of active users, uniformly distributed among dispatchable energy producers, energy storers, and dispatchable energy producers-storers: $n = \{1,\ldots,N/3\} \in \mathcal{N}_{\mathcal{G}\cap\mathcal{S}}, n = \{N/3+1,\ldots,2N/3\} \in \mathcal{N}_{\mathcal{S}\setminus\mathcal{G}}, n = \{2N/3+1,\ldots,N\} \in \mathcal{N}_{\mathcal{G}\setminus\mathcal{S}}$, and $P = 1000 - N$, with $N = 60, N = 120$, and $N = 240$, which correspond to having 6%, 12%, and 24% of active users, respectively. We assign each demand-side user $n \in \mathcal{D}$ the same energy consumption curve, with daily average of $\sum_{h=1}^{24} e_n(h) = 12$ kWh.

Fig. 4 compares the aggregate loads $L(h)$ and the final grid prices resulting from Algorithm 1 at each hour h for the aforementioned percentages of active users. From Fig. 4(a) we can see that, as N increases, the increment in the overall production and storage capacity of the grid allows the demand curve to be progressively more flattened, raising the load during valley hours and shaving off the peak of the consumption. In the specific, the PAR decreases from its initial value 1.5253 to 1.4202 (i.e., 6.9% less) with $N = 60$, to 1.3591 (i.e., 10.9% less) with $N = 120$, and to 1.2653 (i.e., 17.1% less) with $N = 240$. Likewise, the price curve in Fig. 4(b) follows a similar trend, producing a more uniform price per unit of energy throughout the 24 h. In particular, the average grid price per kWh reduces to 0.1349 £/kWh (i.e., 4.5% less) with $N = 60$, to 0.1298 £/kWh (i.e., 8.1% less) with $N = 120$, and to 0.1179 £/kWh (i.e., 16.5% less) with $N = 240$.

V. Conclusion

In this paper, we propose a general grid model that accommodates distributed energy production and storage. In particular, we formulate the day-ahead grid optimization problem, whereby each active user on the demand-side selfishly minimizes his cumulative monetary expense for buying/producing his energy needs, using a game theoretical approach, and we study the existence of the Nash equilibria. We describe a distributed and iterative algorithm based on the proximal decomposition, which allows to compute the optimal strategies of the users with minimum information exchange between the central unit and the demand-side of the network. Simulations on a realistic situation employing practical cost functions show that the demand curve resulting from optimization is sensibly flattened, reducing the need for carbon-intensive and expensive peaking power plants. Finally, it is worth mentioning that the approach presented here, being directly applicable to end users like households and small businesses, can also be extended to larger contexts, such as small communities or cities. In fact, flattening the energy demand along time is clearly beneficial at any layer or scale of the energy grid.

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