

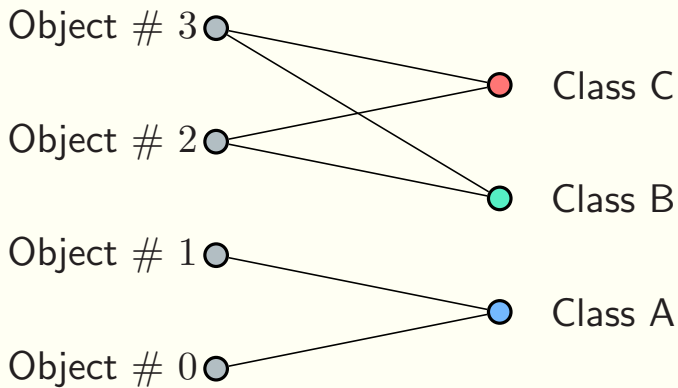
# Learning Bipartite Graphs: Heavy Tails and Multiple Components

a talk by

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# Undirected Weighted Bipartite Graphs



**Goal: learn a Laplacian matrix  
of a bipartite graph from data**

# State-of-the-art Methods

## ❖ Bipartite Structure<sup>1</sup>

$$\begin{aligned} & \underset{B, V \in \mathbb{R}^{p \times k}}{\text{minimize}} && \|B - A\|_F^2 + \eta \text{tr}(V^\top L V), \\ & \text{subject to} && B \geq 0, B \mathbf{1}_q = \mathbf{1}_r, V^\top V = I_k, \end{aligned}$$

## ❖ Spectral Regularization<sup>2</sup>

$$\begin{aligned} & \underset{w \geq 0, V, U, \psi, \lambda}{\text{minimize}} && \text{tr}(\mathcal{L} w S) - \log \det^*(\mathcal{L} w) \\ & && + \frac{\gamma}{2} \left\| \mathcal{A} w - U \text{Diag}(\psi) U^\top \right\|_F^2 + \frac{\beta}{2} \left\| \mathcal{L} w - V \text{Diag}(\lambda) V^\top \right\|_F^2, \\ & \text{subject to} && U^\top U = I, U \in \mathbb{R}^{p \times p}, \psi \in C_\psi, V^\top V = I, V \in \mathbb{R}^{p \times p}, \lambda \in C_\lambda, \end{aligned}$$

where  $C_\lambda$  and  $C_\psi$  are convex sets that define the eigenvalues of the Laplacian and adjacency matrices of  $k$ -component bipartite graphs

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<sup>1</sup>Nie, F. *et al.* Learning a Structured Optimal Bipartite Graph for Co-Clustering, NeurIPS 2017.

<sup>2</sup>Kumar, S. *et al.* A Unified Framework for Structured Graph Learning via Spectral Constraints, JMLR 2020.

# Proposed Formulation

- ❖  $k$ -component, bipartite, student- $t$  graph learning:

$$\underset{\mathbf{L} \succeq \mathbf{0}, \mathbf{B}}{\text{minimize}} \quad \frac{p + \nu}{n} \sum_{i=1}^n \log \left( 1 + \frac{h_i + \text{tr}(\mathbf{B}\mathbf{G}_i)}{\nu} \right) - \log \det^* (\mathbf{L}),$$

$$\text{subject to} \quad \mathbf{L} = \begin{bmatrix} \mathbf{I}_r & -\mathbf{B} \\ -\mathbf{B}^\top & \text{Diag}(\mathbf{B}^\top \mathbf{1}_r) \end{bmatrix}, \text{rank}(\mathbf{L}) = p - k, \mathbf{B} \geq \mathbf{0}, \mathbf{B}\mathbf{1}_q = \mathbf{1}_r.$$

- ❖ we employ the alternating direction method of multipliers (ADMM) and majorization-minimization (MM) to find stationary points of the proposed optimization problems
- ❖ see our supplementary material for convergence proofs 😊

# Datasets and Benchmark Algorithms

## Datasets (Log-returns)

- ❖ US Stock Market ( $r = 333$  S&P500 stocks  $q = 8$  S&P Sector Indices, from Jan. 5th 2016 to Jan. 5th 2021,  $n = 1291$  daily observations)

## Benchmark Models

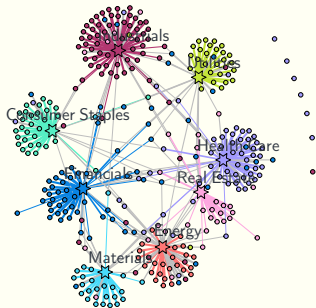
- ❖ Bipartite structure: SOBG<sup>3</sup>
- ❖ Spectral regularization method: SGLA<sup>4</sup>

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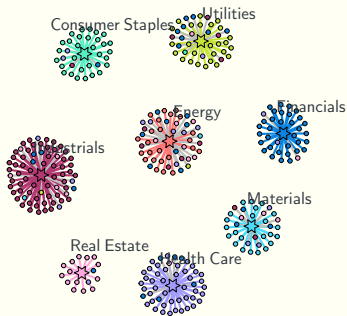
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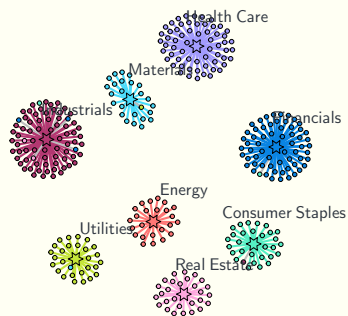
# Learned Graphs



(a) SGLA, accuracy = 0.77,  
modularity = 0.56

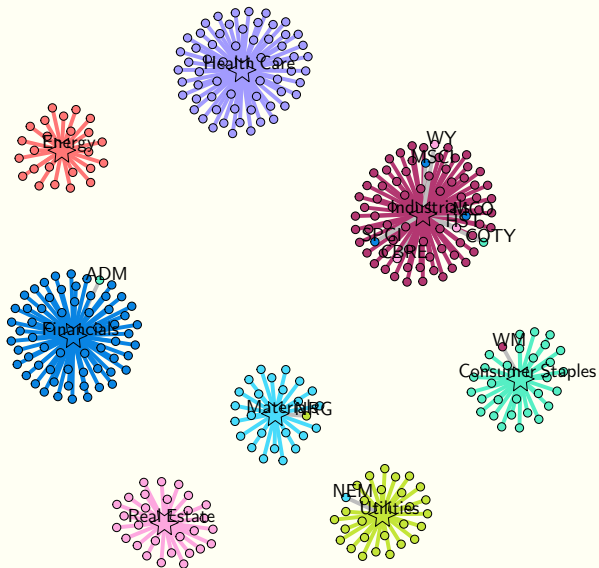


(b) SOBG, accuracy = 0.75,  
modularity = 0.61






(c) proposed, accuracy = 0.97,  
modularity = 0.82

# Learned Graphs





# Thank You!!

- ❖  <https://github.com/convexfi/bipartite>
- ❖  <https://mirca.github.io>
- ❖  @mircaze